Numerical Simulation of Crack Detection in High-Temperature Superconducting Film by Using Contactless Methods

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***Abstract*—The applicability of the scanning permanent magnet method to the crack detection in a high-temperature superconducting (HTS) film is investigated numerically. To this end, a numerical code is developed for analyzing the shielding current density in an HTS film with a crack. In the code, the virtual voltage method is applied to the solution of the initial-boundary-value problem of the shielding current density. The results of computations show that the crack position can be accurately determined by scanning an HTS film in two opposite directions.**

***Index Terms*—Critical current density, high temperature superconductors, integrodifferential equations, surface cracks.**

I. Introduction

As is well known, the critical current density $j_{\rm C}$ is one of the most important parameters for engineering applications of high-temperature superconducting (HTS) films. In this sense, contactless methods have been desired for measuring $j_{\rm C}$ of HTS films.

Ohshima *et al.* [1], [2] proposed the permanent-magnet (PM) method for measuring $j_{\rm C}$ of an HTS film. While bringing a permanent magnet closer to an HTS film, they measured the electromagnetic force acting on the film. Consequently, they found that the maximum repulsive force $F_{\rm M}$ is roughly proportional to $j_{\rm C}$. This tendency implies that $j_{\rm C}$ can be estimated from the measured value of $F_{\rm M}$. This is the basic idea of the PM method.

Although the PM method can be applied not only to the $j_{\rm C}$-measurement [1] but also to the crack detection [2], it is extremely time-consuming. This is mainly because $F_{\rm M}$ must be determined at each measurement point. For the purpose of resolving this problem, Hattori *et al.* [3] have improved the PM method. In the improved method, the magnet is moved along the film surface and, simultaneously, the electromagnetic force F_z acting on the film is measured. As a result, the $j_{\rm C}$-distribution can be successfully determined from the measured F_z-distribution. Throughout the present study, the improved method is called the scanning PM method. However, it is not clear whether or not cracks can be detected by using the scanning PM method.

The purpose of the present study is to numerically investigate the applicability of the scanning PM method to the crack detection in an HTS film.

II. Governing Equation

In the scanning PM method, a cylindrical permanent magnet of radius R is moved along the surface of an HTS film of thickness b. In terms of the Cartesian coordinate system, the symmetry axis of the permanent magnet is expressed as $(x, y) = (x_{\rm A}, y_{\rm A})$.

We first assume that the HTS film has a rectangular cross section Ω of length l and width w. Moreover, we assume that the film contains a crack whose cross section is a line segment connecting two points $(x_{\rm c}, y_{\rm c} \pm L_{\rm c}/2)$ in the xy plane. In other words, the crack of length $L_{\rm c}$ is assumed to be parallel to y-axis. For this case, the boundary $\partial\Omega$ of Ω consists of the outer boundary C_0 and the inner boundary C_1.

Under the thin-plate approximation, there exists a scalar function $S(\boldsymbol{x}, t)$ such that $\boldsymbol{j} = (2/b)(\nabla S \times \boldsymbol{e}_z)$ and its time evolution is governed by the following equation [4]:

$$\mu_0 \partial_t(\hat{W} S) + \boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{E}) = -\partial_t \langle \boldsymbol{B} \cdot \boldsymbol{e}_z \rangle. \qquad (1)$$

Here, $\boldsymbol{B}/\mu_0$ and $\boldsymbol{E}$ are the applied magnetic field and the electric field, respectively, and $\langle\ \rangle$ denotes an average operator over the thickness. In addition, $\hat{W}$ is the operator defined by

$$\hat{W} S \equiv \frac{2S(\boldsymbol{x}, t)}{b} + \iint_{\Omega} Q(|\boldsymbol{x} - \boldsymbol{x}'|)\, S(\boldsymbol{x}', t)\, d^2\boldsymbol{x}',$$

where $Q(r) = -(\pi b^2)^{-1}[r^{-1} - (r^2 + b^2)^{-1/2}]$. As the J-E constitutive relation, the power law [5]-[7] is adopted.

The boundary conditions to (1) are assumed as follows:

$$S = 0 \ \text{on} \ C_0, \quad \frac{\partial S}{\partial s} = 0 \ \text{on} \ C_1, \quad \oint_{C_1} \boldsymbol{E} \cdot \boldsymbol{t}\, ds = 0,$$

where s is an arclength along C_1 and $\boldsymbol{t}$ denotes a unit tangential vector on C_1. The initial condition to (1) is assumed as follows: $S = 0$ at $t = 0$.

By solving the initial-boundary-value problem of (1), we can determine the time evolution of the shielding current density. Throughout the present study, the physical and geometrical parameters are fixed as follows: $l = 30$ mm, $w = 10$ mm, $b = 1$ μm, $j_{\rm C} = 1.5$ MA/cm^2, and $y_{\rm A} = y_{\rm c} = 0$ mm.

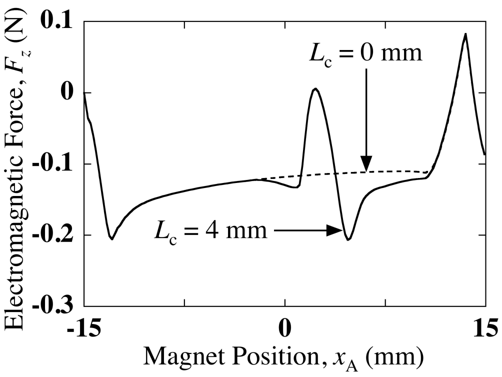

Figure 1. Dependences of the electromagnetic force F_z on the magnet position x_A for the case with $x_c = 3$ mm and $x_A(t) = x_+(t)$.

III. NUMERICAL RESULTS

On the basis of the virtual voltage method [4], a numerical code has been developed for analyzing the shielding current density in an HTS film. In this section, the influence of a crack on the scanning PM method is assessed by means of the code. In the PM method, an HTS film is scanned with a permanent magnet in two directions. Specifically, the movement of the magnet are assumed as $x_A(t) = \pm(vt - l/2) \equiv x_\pm(t)$, where v is the magnitude of the scanning velocity. Throughout the present study, v is fixed as $v = 2$ mm/s.

Let us first investigate the influence of a crack on the electromagnetic force F_z acting on the film. Dependences of F_z on x_A are shown in Fig. 1. This figure indicates that, for $-2 \text{ mm} \lesssim x_A \lesssim 11$ mm, two F_z-x_A curves overlap with each other. Otherwise, two curves show quite different behaviors. This result suggests that the force difference $\mathit{\Delta} F_z(L_c)(\equiv F_z(L_c) - F_z(0))$ changes violently near the crack.

Dependences of $\mathit{\Delta} F_z$ on x_A are numerically determined for two types of the magnet movement and are depicted in Fig. 2. We see from this figure that two $\mathit{\Delta} F_z$-x_A curves intersect with each other. If x_A-coordinate of the intersection point is denoted by x_c^*, we get $x_c^* = 3.01$ mm for the case with Fig. 2. Note that the crack position is assumed as $x_c = 3$ mm for this case. Thus, the estimated value x_c^* of the crack position agrees well with its assumed value x_c. This result indicates that the crack position can be determined by scanning an HTS film in two opposite directions.

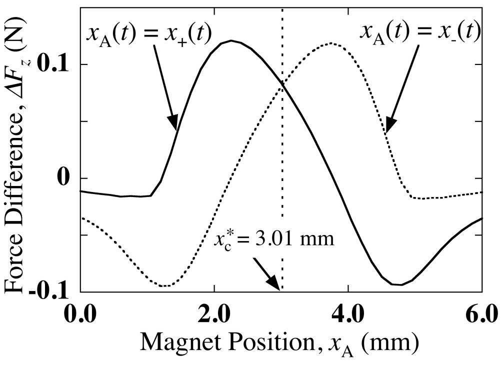

Figure 2. Dependences of the force difference $\mathit{\Delta} F_z$ on the magnet position x_A for the case with $x_c = 3$ mm and $L_c = 4$ mm.

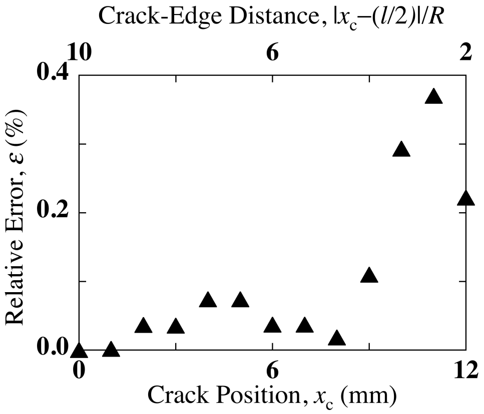

Figure 3. Dependence of the relative error ε on the crack position x_c for the case with $L_c = 4$ mm.

Next, we investigate the accuracy of the crack position estimated by the above method. As a measure of the accuracy, we use the relative error defined by $\varepsilon \equiv |x_c^* - x_c|/l$. The relative error is evaluated as a function of the crack position and is depicted in Fig. 3. We see from this figure that the relative error ε takes only 0.5% or less.

From the above results, we can conclude that the crack position can be accurately detected by scanning an HTS film in two opposite directions. Incidentally, for the case where the crack-edge distance is small enough to satisfy $|x_c - l/2| < 2R$, the crack position cannot be determined at all. This is mainly because two $\mathit{\Delta} F_z$-x_A curves never intersect with each other.

REFERENCES

[1] S. Ohshima, K. Takeishi, A. Saito, M. Mukaida, Y. Takano, T. Nakamura, I. Suzuki, and M. Yokoo, “A simple measurement technique for critical current density by using a permanent magnet,” *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 2911-2914, Jun. 2005.

[2] S. Ohshima, K. Umezu, K. Hattori, H. Yamada, A. Saito, T. Takayama, A. Kamitani, H. Takano, T. Suzuki, M. Yokoo, and S. Ikuno, “Detection of Critical Current Distribution of YBCO-Coated Conductors Using Permanent Magnet Method, ” *IEEE Trans. Appl. Supercond.*, vol. 21, no. 3, pp. 3385-3388, Jun. 2011.

[3] K. Hattori, A. Saito, Y. Takano, T. Suzuki, H. Yamada, T. Takayama, A. Kamitani, and S. Ohshima, “Detection of smaller J_c region and damage in YBCO coated conductors by using permanent magnet method,” *Physica C*, vol. 471, no. 21-22, pp. 1033-1035, Nov. 2011.

[4] A. Kamitani, T. Takayama, A. Saitoh, and H. Nakamura, ”Accurate and stable numerical method for analyzing shielding current density in high-temperature superconducting film containing cracks,” *Plasma Fusion Res.*, vol. 7, 2405024, May 2012.

[5] R. Fresa, G. Rubinacci, S. Ventre, F. Villone, and W. Zamboni, ”Fast solution of a 3-D integral model for the analysis of ITER superconducting coils,” *IEEE. Trans. Magn.*, vol. 45, no. 3, pp. 988-991, Mar. 2009.

[6] A. Kameni, J. Lambrechts, J. F. Remacle, S. Mezani, F. Bouillault, and C. Geuzaine, ”Discontinuous Galerkin method for computing induced fields in superconducting materials,” *IEEE. Trans. Magn.*, vol. 48, no. 2, pp. 591-594, Feb. 2012.

[7] R. Brambilla, F. Grilli, and L. Martini, ”Integral equations for computing AC Losses of radially and polygonally arranged HTS thin tapes,” *IEEE Trans. Appl. Supercond.*, vol. 22, vol. 4, 8401006, Aug. 2012.