Fast Technique for Lorentz Force Calculations in Nondestructive Testing Applications

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***Abstract*—The primary aim of this work is to present a new 2D/3D numerical technique providing fast calculation of Lorentz forces acting on a permanent magnet moving relatively to a solid electrically conducting object. This specific field configuration represents a typical problem arising in novel Lorentz force eddy current testing applications. The proposed technique is based on several model simplifications which enable considerable reduction of the total simulation time while maintaining the accuracy of the solution. For evaluation of the computational requirements and verification of the obtained results the well-established sliding mesh technique (SMT) has been used. The full paper will comprise the experimental validation of the obtained results as well.**

***Index Terms*—Eddy currents, finite element methods, Lorentz force, nondestructive testing**

I. Introduction

In the field of nondestructive testing and evaluation (NDT&E) of materials there is a strong demand on the development of new NDT&E techniques which can increase the reliability and maintain the required safety of the product. A recent technique, referred to as Lorentz force eddy current testing (LET), exploits the advantages of applying DC magnetic fields and relative motion providing deep and relatively fast testing of electrically conducting materials [1], [2]. In principle, LET represents a modification of the traditional eddy current testing from which it differs in two aspects, namely (i) how eddy currents are induced and (ii) how their perturbation is detected. In LET eddy currents are generated by providing the relative motion between the conductor under test and a permanent magnet (Fig. 1). If the magnet is passing by a defect, the Lorentz force acting on it shows a distortion whose detection is the key for the LET working principle. If the object is free of defects, the resulting Lorentz force remains constant [2]. Due to its complex geometrical configurations the computational analysis of LET systems is strictly limited to numerical techniques [3], [2], [5]. Moreover, due to its flexibility regarding the model discretization the finite element method (FEM) is commonly applied. Thus, the particular emphasis is placed on the development of effective FEM-based techniques providing the reduction of the overall computational requirements while maintaining the accuracy of the solution. In this paper, we present the weak reaction approach (WRA) that allows fast and low-cost simulations of 2D/3D LET systems. WRA is based on the assumption that the reaction magnetic field of the induced currents, denoted as $\mathbf{b}$, is small compared to the applied magnetic field of the magnet $\mathbf{B}_0$ ($\mathbf{b} \ll \mathbf{B}_0$). As it will be shown below, this assumption is fully justified when the so-called magnetic Reynolds number (R_m), defined as $R_m = v\sigma\mu h_c/2$ is low, i.e. $R_m \ll 1$ [6], [7] (Fig. 1).

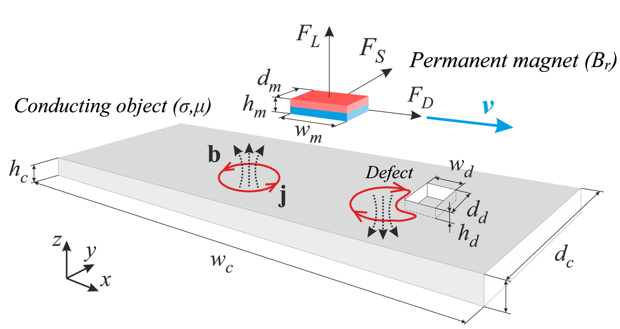


Figure 1: LET working principle.

II. Weak Reaction Approach

Assuming the frame of reference in which the magnet is stationary, i.e. the conductor moving in the opposite direction with velocity $-\mathbf{v}$ (Fig. 2), LET systems are fully described by the transient magnetic field diffusion-convection equation [4], [5]. In the non-dimensional form this equation is described as follows [7]

$$\tilde{\nabla} \times (\tilde{\nabla} \times \tilde{\mathbf{B}}) = -R_m \left(\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} + \tilde{\nabla} \times (\tilde{\mathbf{v}} \times \tilde{\mathbf{B}}) \right), \tag{1}$$

where "∼" indicates the non-dimensional quantities and $\tilde{\mathbf{B}} = \tilde{\mathbf{B}}_0 + \tilde{\mathbf{b}}$ is the total magnetic field. When R_m is small (1) simplifies to $\tilde{\nabla} \times (\tilde{\nabla} \times (\tilde{\mathbf{B}}_0 + \tilde{\mathbf{b}})) = 0$, which is only fulfilled when $\tilde{\mathbf{b}} = \mathbf{0}$. This in fact represents the main idea of the WRA. In this case only the conducting region needs to be modelled (Fig. 2). The electric potential V and induced eddy currents are described by the Ohm's law for moving conductors

$$\mathbf{j} = \sigma(-\nabla V + \mathbf{v} \times \mathbf{B}_0). \tag{2}$$

Satisfying the current conservation law $\nabla \cdot \mathbf{j} = 0$, in low-$R_m$ range, LET systems are described simply by the Laplace equation

$$\nabla^2 V = \nabla \cdot (\mathbf{v} \times \mathbf{B}_0) = \mathbf{B}_0 \cdot (\nabla \times \mathbf{v}) - \mathbf{v} \cdot (\nabla \times \mathbf{B}_0) = 0. \tag{3}$$

The specific value of V is obtained by specifying the boundary conditions for the given problem. The boundary conditions are obtained by making the normal components of the induced current density on the walls of the conductor and defects to

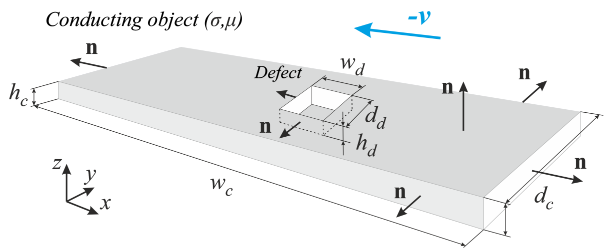


Figure 2: Implementation of the WRA.

Table I: Geometrical and material properties

$w_m \times h_m \times d_m$ [mm]	$w_c \times h_c \times d_c$ [mm]	$w_d \times h_d \times d_d$ [mm]	σ [MS/m]	B_r [T]
$15 \times 25 \times 15$	$250 \times 50 \times 50$	$12 \times 2 \times 2$	30.61	1.17

Table II: Computational requirements and simulation time

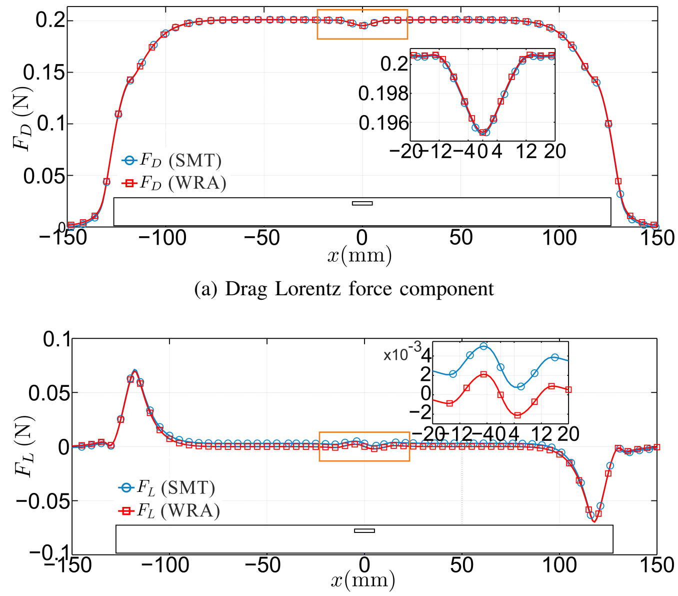
Method	Solv.	NoE	DoF	RAM	Sim. time
SMT	Direct	257 343	922 110	~ 20.1 GB	18h : 28min
WRA	Iterative	89 430	132 499	~ 0.73 GB	26min : 10s

zero, i.e. $\mathbf{n} \cdot \mathbf{j} = 0$ (Fig. 2) [7]. In the numerical implementation of the WRA there are two possibilities to prescribe the primary magnetic field $\mathbf{B}_0$. If the used permanent magnets have simple shapes, like of a sphere or a rectangle, $\mathbf{B}_0$ can be obtained in the closed-form analytically [8]. This approach has been used here, where the expression for $\mathbf{B}_0$ is not shown for simplicity. For complex magnet systems, $\mathbf{B}_0$ can be determined numerically. However, this requires time taking interpolation of the field resulting in longer simulation time.

III. Results and Discussion

The basic LET configuration under investigation is shown in Fig. 1. The rectangular permanent magnet described by the magnetization vector $\mathbf{M} = B_r/\mu_0 \mathbf{e}_z$ is moving with constant velocity $\mathbf{v}$ and lift-off distance dz relatively to a conductor containing an artificial defect placed centrally at depth d (Fig. 2). All material properties and geometrical parameters used for the analysis are summarized in Table I. Due to the symmetry of the model with respect to the $X0Z$-plane, the side component (F_S) of the Lorentz force vanishes, i.e. only the drag (F_D) and the lift (F_L) force components are used for comparison. As a reference method for WRA the well-established SMT has been used [9], [10]. We adopt SMT for modelling linear displacement according to the given LET problem (Fig. 1). Thus, the computational domain is divided into two independent parts, namely, the stationary (conductor) and the moving part (magnet). The coupling interface is placed in the air region [5].

Even for moderate values of R_m (e.g. $R_m = 0.1$) a very good agreement between SMT and WRA for both Lorentz force components can be observed (Fig. 3). However, it should be mentioned that due to weak reaction ($\mathbf{b} = 0$) the lift component of the Lorentz force for a defect-free conductor is zero. Nevertheless, WRA accurately describes the perturbation of the Lorentz force due to defects. This is of main interest in LET systems. The computational costs resulted using the SMT and WRA are summarised in Table II. A considerable reduction in



(a) Drag Lorentz force component

(b) Lift Lorentz force component

Figure 3: Comparison of the Lorentz force perturbations obtained using SMT and WRA for $dz = 1$ mm, $d = 2$ mm and $R_m = 0.1$.

the computational requirements and total simulation time has been obtained.

IV. Conclusions

A new WRA is proposed that permits fast analysis of LET problems. WRA has been successfully applied to solve the given 3D LET configuration showing a good agreement with the well-established sliding mesh technique in the range of small R_m values. The obtained reduction in the computational costs and modelling time is considerable.

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