Improvement of Unified Boundary Integral Equation Method in Magnetostatic Shielding Analysis

K. Ishibashi[1,3], T. Yoshioka[2], S. Wakao[2], Y.Takahashi[4], Z. Andjelic[3] and K. Fujiwara[4]
[1]2-55-12-505 Sangenjaya, Setagaya, Tokyo 154-0024, Japan
[2]Department of Electrical Engineering and Bioscience, Waseda University, Tokyo 169-8555, Japan
[3]POLOPT Technologies GmbH, Bruggerstrasse 44, 5400 Baden, Switzerland
[4]Department of Electrical Engineering, Doshisha University, Kyoto 610-0321, Japan
[1]yui@g03.itscom.net

Abstract — A boundary integral equation (BIE) with double layer charge as unknown has been applied to generic magnetostatic problems. However, in applying the BIE to solve shielding problems, as the permeability becomes higher, computing accuracy in the shielded area becomes worse and worse. In order to overcome the problem, we propose a new approach utilizing a concept of the boundary element method.

I. Introduction

We have direct and indirect approaches to derive boundary integral equations (BIEs) to solve magnetostatic problems. The direct approach gives BIEs with potential φ_H and its normal derivative H_n as unknowns. This approach is called usually Boundary Element Method (BEM) [1]. As discussed in [2], the BIEs of BEM are applicable to generic problems without cancellation errors but have cut-surface problems due to current sources. The indirect approach gives a BIE with single layer charge σ_s as unknown [2-4], which is called usually Surface Charge Method (SCM). Also, it gives another BIE with double layer charge σ_d as unknown [5-7], which is called here indirect BIE. Since both of the BIEs derived by the indirect approach contain only one unknown, they are advantageous to the BIEs of BEM from the viewpoint of numerical analysis. However, the SCM has deficiency such as cancellation errors, and the indirect BIE has also cut-surface problems as BEM. It has been proposed how to avoid the cancellation errors in the SCM [2-4] and the cut-surface problems in the indirect BIE [6, 7]. Therefore, these approaches have been capable of solving generic problems, but the SCM is incapable of treating magnetic fields at edges and corners while the indirect BIE is capable of doing them [5]. When we adopt the indirect BIE to analyze shielding problems with the high permeability, we cannot get accurate results in evaluating fields in shielded areas. In this paper, we study how to overcome the deficiency utilizing a concept of BEM.

II. Derivation of Indirect BIE

The indirect approach replaces a magnetic material with the magnetic permeability μ to the magnetization $\boldsymbol{M}$, which gives an integral form of magnetic field $\boldsymbol{H}$. Ampere's law relates $\boldsymbol{M}$ with an equivalent current $\boldsymbol{J}_v$ defined as: $\boldsymbol{J}_v = \nabla\times\boldsymbol{M}$. The difference of μ on each side of interface produces a surface current $\boldsymbol{J}_s$ defined as: $\boldsymbol{J}_s = \boldsymbol{n}\times\boldsymbol{M}$ with the unit normal $\boldsymbol{n}$. If μ is constant, $\boldsymbol{J}_v$ is zero and the material is regarded as replaced to $\boldsymbol{J}_s$, which gives an integral form of magnetic flux density $\boldsymbol{B}$. The current gives the solenoidal field and it cannot give the scalar potential φ_B. In order to derive φ_B, we introduce loop currents J_l for $\boldsymbol{J}_s$ as shown in Fig. 1. The concept of magnetic shell relates J_l with σ_d as: $J_l=\sigma_d$ [8]. By virtue of σ_d, φ_B at any point in the whole space is given as:

$$\varphi_B(P)=\int_{S_i+S_o} J_l \frac{\boldsymbol{n}_i\cdot\boldsymbol{r}}{4\pi r^3}dS+\mu_0\varphi_{He}(P) \tag{1}$$

where (P) denotes values at the observation point P_o, S_i and S_o are the inner and outer surfaces S of shielding material, $\boldsymbol{n}_i$ is the unit normal at the integration point P_i on S, $\boldsymbol{r}$ is the distance from P_i to P_o, and φ_{He} is the excitation potential.

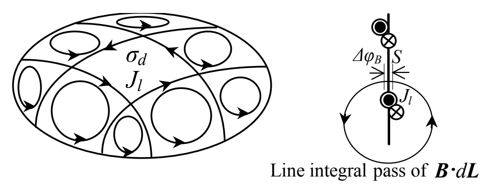

Fig. 1 Magnetic shell and a part of its cross-section.

As the difference $\Delta\varphi_B$ in φ_B either side of S is J_l as shown in Fig. 1, the line integral of $\boldsymbol{B}\cdot d\boldsymbol{L}$ about any closed path L piercing S is zero. Taking the gap J_l into account, obtaining φ_B on each side of S and enforcing the boundary condition of the tangential component H_t of $\boldsymbol{H}$ on S, we get a BIE as:

$$\frac{\Omega_p\mu_r-\Omega_p+4\pi}{4\pi(\mu_r-1)}J_l(P)+\int_{S_i+S_o} J_l \frac{\boldsymbol{n}_i\cdot\boldsymbol{r}}{4\pi r^3}dS=-\mu_0\varphi_{He}(P) \tag{2}$$

where Ω_p is the solid angle subtended at P_o, and μ_r is the relative permeability [5]. As the thickness of material approaches zero, J_l facing each other on smooth surfaces is becoming the same and (2) can be modified as its degree of freedoms becomes half. The boundary condition of normal component B_n of $\boldsymbol{B}$ is automatically fulfilled because J_l doesn't produce any gap of B_n on S. Since φ_{He} produced by current sources is multi-valued, the cut-surface S_c [8] is utilized to get the unified BIE to solve generic problems [7].

Once J_l has been obtained, $\boldsymbol{B}$ is given by the negative gradient of φ_B in (1) and the integral form of $\boldsymbol{B}$ is given by adding $\boldsymbol{B}_\sigma$ due to J_l and $\boldsymbol{B}_e$ due to source currents $\boldsymbol{I}_c$ as:

$$\boldsymbol{B}(P)=\boldsymbol{B}_\sigma(P)+\boldsymbol{B}_e(P)=\sum_{N_s} J_l\oint_{\Delta L}\frac{\boldsymbol{u}_J\times\boldsymbol{r}}{4\pi r^3}dL+\mu_0\frac{\boldsymbol{I}_c\times\boldsymbol{r}}{4\pi r^3} \tag{3}$$

where N_s is the total number of segmental elements on S, $\boldsymbol{u}_J$ is the direction of J_l, which circulates anticlockwise along the contour of the element and ΔL is the length of J_l [5].

III. Numerical Analysis of Magnetostatic Shield

The solution J_l of indirect BIE in (2) gives correctly H_t evaluated by (3) because (2) has been derived from the condition of continuity of H_t. Even though B_n evaluated by (3) is continuous on S, B_n is theoretically incorrect when the theoretical B_n is much weaker than the normal component B_{en} of $\boldsymbol{B}_e$ because the continuity of B_n is fulfilled indirectly as described previously. As μ_r of shielding material increases, B_n on S_i becomes much weaker than B_{en} and the calculation of $\boldsymbol{B}$ within S_i suffers cancellation errors as observed in the SCM. In order to get accurate $\boldsymbol{B}$ within S_i, we propose an approach utilizing a concept of BEM.

Green's theorem gives the scalar potential φ_H at an observation point P_o bounded by a closed surface S_i, where φ_{He} =0, as:

$$\varphi_H(P) = \int_{S_i} \frac{H_n}{4\pi r} dS + \int_{S_i} \varphi_H \frac{\boldsymbol{n}_i \cdot \boldsymbol{r}}{4\pi r^3} dS \tag{4}$$

where H_n is normal derivative of φ_H [8]. The potential φ_H on S_i is related to J_l as:

$$\varphi_H = J_l \big/ (\mu - \mu_0). \tag{5}$$

Substituting σ_H in (5) for that in (4) and obtaining φ_H on S_i in the shielded area, we get a BIE to solve H_n as:

$$\int_{S_i} H_n \frac{1}{4\pi r} dS = \frac{1}{\mu - \mu_0}\left[\frac{J_l(P)}{2} - \int_{S_i} J_l \frac{\boldsymbol{n}_i \cdot \boldsymbol{r}}{4\pi r^3} dS\right]. \tag{6}$$

The magnetic field $\boldsymbol{H}_i$ within S_i is given by the negative gradient of φ_H in (4). Once H_n has been obtained, $\boldsymbol{H}_i$ at any point P_o in the shielded area is given as:

$$\boldsymbol{H}_i(P) = -\nabla\varphi_H(P) = \sum_{N_i}\int_{\Delta S} \frac{H_n \boldsymbol{r}}{4\pi r^3} dS + \sum_{N_i} \frac{J_l}{\mu - \mu_0} \oint_{\Delta L} \frac{\boldsymbol{u}_J \times \boldsymbol{r}}{4\pi r^3} dL \tag{7}$$

where N_i is the total number of segmental elements on S_i, ΔS is the area of the element.

IV. Numerical Validation of Proposed Approach

We solve a shielding problem with a magnetic hollow sphere (Inner radius: R_i=4.95 cm, Outer radius: R_o=5 cm) in the uniform magnetic field $\boldsymbol{H}_e$ of 1 A/cm. The origin of the coordinate system is located at the sphere center. The direction of $\boldsymbol{H}_e$ is parallel to the Z-axis, and μ_r=10000.

Utilizing the model's symmetry, we treat the problem as two-dimensional one. Dividing azimuthally both of the inner and outer surfaces, S_i and S_o, into N segments and adopting the linear element, we obtain J_l in (2) and evaluate $\boldsymbol{B}$ in (3). The computed results are shown in Fig. 2(a) and (b), where the symbol Δ and ▽ denote the computed values when N=11 and 21, respectively, and the solid line gives the theoretical value [9]. The accuracy of computed results within S_i (shielded area) becomes worse as the meshes become rougher. After we apply the approach explained in 'Chapter III', the computed results in the shielded area are improved significantly. The improved values are shown by the blacken symbols, which are seen almost on the line of theoretical values.

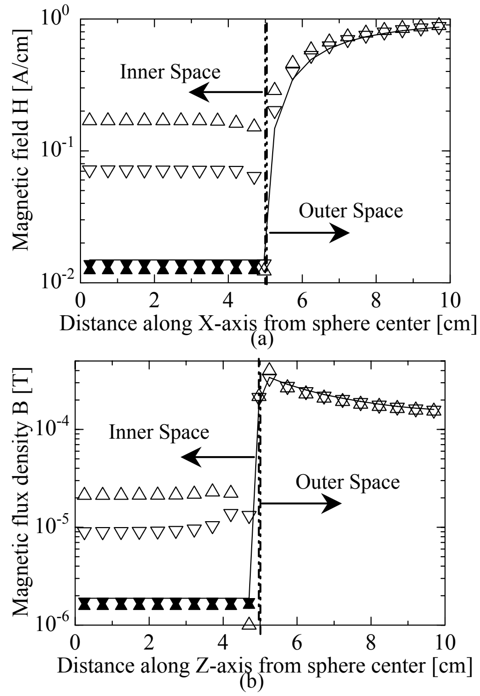

Fig. 2. Computed results of magnetic field $\boldsymbol{H}$ and flux density $\boldsymbol{B}$.

V. Conclusions

When we apply the indirect BIE to solve shielding problems, we can't get accurate results in the shielded area due to cancellation errors. We have proposed an approach to overcome the deficiency utilizing a concept of BEM and confirmed that the approach works effectively, and also the technique of this approach is applicable to the SCM. Though we have to solve the additional BIE derived by BEM concept, it is not time-consuming to solve it because H_n on S_i facing the shielded area only has to be solved.

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