Global sensitivity analysis of magneto-electric sensor model parameters

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***Abstract*—This paper deals with the global sensitivity analysis of magnetoelectric devices model for magnetic field sensing. The magnetoelectric sensor response is represented by a 2D finite element model. Several material parameters of the model are assumed to be uncertain. A non intrusive spectral projection method is used to quantify the variability of the model outputs. Using the Sobol indices, the most influential parameters on the magnetoelectric sensor sensitivity are identified.**

I. INTRODUCTION

Magnetic field sensors based on extrinsic magnetoelectric (ME) effect detect static magnetic field by measuring a harmonic electric voltage [1]. Magnetic field sensors consist of magnetostrictive-piezoelectric multilayers. Piezoelectric materials properties are provided by manufacturers with reasonable accuracy. However, the randomness on magnetostrictive materials properties can arise from manufacturing process deviation or lack of quality controls. The main properties of magnetostrictive materials are the permeability, the magnetostrictive coefficient and the Lamé coefficients related respectively to magnetic, magneto-elastic and elastic properties. These uncertain parameters are modeled as random variables. Previous papers have described the finite element model of such sensor [2] [3] without taking into account the uncertainties in the material properties. In this communication, we propose a stochastic model of magnetoelectric sensors with uncertain material parameters. In order to solve stochastic problem, the Non Intrusive Spectral Projection (NISP) method is employed. This method is compared with the Monte Carlo Simulation Method (MCSM). Finally the Sobol indices are calculated in order to determine the most influential parameters.

II. ME SENSOR FINITE ELEMENT MODEL

The extrinsic ME effect results from the combination of magnetostrictive and piezoelectric effects. Previous papers have shown that the performance of the device is greatly improved under dynamic excitation [1]. Indeed, the ME coefficient takes advantage of the superimposition of a small harmonic magnetic field h_{ac} at mechanical resonance frequencies of sensor and a static magnetic field $\mathbf{H}_{dc}$. Higher sensitivity of ME sensor has been obtained due to the non linearity of the magnetostrictive behavior. The constitutive laws of piezoelectric are assumed to be linear. The magnetostrictive constitutive laws are non linear. For the magnetostrictive material, the total strain $\mathbf{S}$ is divided into the elastic strain $\mathbf{S}^e$ and the magnetostriction strain $\mathbf{S}^\mu$, $\mathbf{S} = \mathbf{S}^e + \mathbf{S}^\mu$. Then the magnetostrictive strain can be written as a function of magnetostrictive coupling coefficient β, magnetic induction $\mathbf{B}$ and magnetization $\mathbf{M}$:

$$s^\mu_{ij} = \frac{\beta}{2}(3b_ib_j - \delta_{ij}||\mathbf{B}||)\frac{||\mathbf{M}||^2}{||\mathbf{B}||^2} \tag{1}$$

The static constitutive laws are given by [2]:

$$\begin{aligned} t_{ij} &= C^{ms}_{ijkl}s_{kl} - t^\mu_{kl} \\ h_i &= \nu_{ij}b_j - \frac{\partial t^\mu_{kl}}{\partial b_i}(s_{kl} - s^\mu_{kl}) \end{aligned} \tag{2}$$

with t_{ij} the stress, t^μ_{ij} the magnetostrictive stress, ν the reluctivity of material. The ME deterministic finite element model consists of two subprograms. In the first subprogram, for each finite element, magnetization $\mathbf{M}^e$ and magnetic induction $\mathbf{B}^e$ are calculated, depending on the imposed magnetic field $\mathbf{H}_{dc}$. Using these values, we can estimate the parameters of magnetostrictive linearized constitutive laws given in equation (3):

$$\begin{pmatrix} \widetilde{\mathbf{T}} \\ \widetilde{\mathbf{H}} \end{pmatrix} = \begin{bmatrix} \widetilde{\mathbf{c}}_{ms} & -\widetilde{\mathbf{q}}^{\mathrm{t}}_{ms} \\ -\widetilde{\mathbf{q}}_{ms} & \widetilde{\nu}_{ms} \end{bmatrix} \begin{pmatrix} \widetilde{\mathbf{S}} \\ \widetilde{\mathbf{B}} \end{pmatrix} \tag{3}$$

where $\widetilde{\mathbf{c}}_{ms}$ is the stiffness tensor as a function of Lamé coefficients μ^* and λ^*. The coupling matrix $\widetilde{\mathbf{q}}_{ms}$ is linked directly to μ^*, β, $\mathbf{B}^e$ and $\mathbf{M}^e$. The equivalent reluctivity $\widetilde{\nu}_{ms}$ is calculated as a function of the permeability μ, $\mathbf{B}^e$ and $\mathbf{M}^e$, and the initial mechanical conditions. We note $\widetilde{X}(\widetilde{a}, \widetilde{b})$ the small variation of X around a polarization point $X_0(a_0, b_0)$. Imposing a harmonic magnetic field h_{ac} at a resonance frequency of ME sensor, the second subprogram, using the constitutive law (3), calculates the harmonic electric voltage v_{ac}. The electric voltage v_{ac} is then linked to the static magnetic field $\mathbf{H}_{dc}$.

III. UNCERTAINTY QUANTIFICATION

Let consider a function $Y(\mathbf{u}(\boldsymbol{\theta}))$ where $\mathbf{u}(\theta)$ is a vector of M independent uniformly distributed random variables in the interval $[-1, 1]$ with a finite variance. The first method to study the function $Y(\mathbf{u}(\boldsymbol{\theta}))$ consists in using the MCSM. This method is very simple and robust but can be very time consuming. Another method consists in using a stochastic spectral approach. The $Y(\mathbf{u}(\boldsymbol{\theta}))$ can be expanded using a Polynomial Chaos Expansion:

$$Y(\mathbf{u}(\boldsymbol{\theta})) = \sum_{i=0}^{\infty} y_i \mathcal{L}_i(\mathbf{u}(\boldsymbol{\theta})) \tag{4}$$

with y_i the real coefficients and $\mathcal{L}_i(\mathbf{u}(\boldsymbol{\theta}))$ the multivariate polynomials generated from product of monovariate Legendre polynomials. Since the multivariate polynomials $\mathcal{L}_i(\mathbf{u}(\boldsymbol{\theta}))$ are orthogonal, the coefficient y_i is given by:

$$y_i = \frac{\mathbb{E}(Y(\mathbf{u}(\boldsymbol{\theta})\mathcal{L}_i(\mathbf{u}(\boldsymbol{\theta})))}{\mathbb{E}(\mathcal{L}_i(\mathbf{u}(\boldsymbol{\theta})^2))} \tag{5}$$

with $\mathbb{E}(\mathbf{X}(\theta))$ the expectation of random variable $\mathbf{X}(\boldsymbol{\theta})$. The NISP method enables to approximate $Y(\mathbf{u}(\boldsymbol{\theta}))$ by truncating (4) and by calculating y_i using the Gauss quadrature scheme [5].

$$y_i = \frac{\sum_{j=1}^{N} Y(\mathbf{u}^j)\mathcal{L}_i(\mathbf{u}^j)\omega^j}{\sum_{j=1}^{N} \mathcal{L}_i^2(\mathbf{u}^j)\omega_j} \tag{6}$$

with $\mathbf{u}^j$ and ω^j the Gauss points and their associated weights, N the number of Gauss points. From the truncated expansion (4), which is equivalent to a stochastic surrogate model, statistical parameters such as the mean and the standard deviation can be easily extracted. To study the influence of the parameters $\mathbf{u}(\boldsymbol{\theta})$ on $Y(\mathbf{u}(\boldsymbol{\theta}))$, Sobol proposed indices based on the decomposition of the variance of $Y(\mathbf{u}(\boldsymbol{\theta}))$ in the form [4]:

$$\mathrm{var}(Y) = \sum_{i=1}^{M} D_i + \sum_{1\leq i<j\leq M} D_{ij} + \ldots + D_{1\ldots M} \tag{7}$$

We denote by $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_K)$ a K-tuple $(K \leq M)$ such that $(\alpha_1 < \ldots < \alpha_K)$ and $\alpha_i \in [1, M]$. $D_{\boldsymbol{\alpha}}$ is the fraction of variance due to the interaction of the inputs $\mathbf{v} = (u_{\alpha_1}, \ldots, u_{\alpha_K})$. The Sobol index $S_{\boldsymbol{\alpha}}$ is defined by: $S_{\boldsymbol{\alpha}} = \dfrac{D_{\boldsymbol{\alpha}}}{\mathrm{var}(Y(\mathbf{u}(\boldsymbol{\theta})))}$.
The Sobol indices can be calculated using the MCSM or from the approximation (4) of $Y(\mathbf{u}(\boldsymbol{\theta}))$. The sum of the Sobol indices is equal to one and they are always positive. The first order Sobol indices S_i enable to evaluate the influence of the input $u_i(\boldsymbol{\theta})$. The higher S_i, the more influent $u_i(\boldsymbol{\theta})$.

IV. Results and Conclusion

The ME sensor is a trilayer structure presented in Fig 1

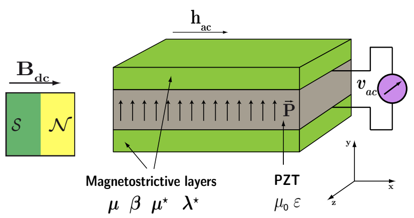

Fig. 1. Magnetoelectric sensor

The magnitude of the voltage v_{ac} obtained between the electrodes of the piezoelectric layer depends on the DC magnetic flux density $\mathbf{B}_{dc}$. The permeability (μ), the magnetostrictive coupling coefficient (β) and Lamé coefficients (μ^* and λ^*) are assumed to be independent and uniformly distributed. The parameter variations have been assumed to be of 20% on β and 5% on the other material parameters (μ, μ^*, λ^*). Tab. 1 gives the mean m and the standard deviation value σ for the considered random variables.

	μ	β	μ^*	λ^*
m	100	2.40E-5	3.85E+10	5.77E+10
σ	8.33	0.76E-6	1.23E+9	2.76E+8

TABLE 1
VALUE AND STANDARD DEVIATION OF RANDOM VARIABLES

In Fig. 2, the harmonic electric voltage v_{ac} is plotted versus the magnetic flux density $\mathbf{B}_{dc}$. The blue points correspond to the mean of the electric voltage obtained by using MCSM. The red points, corresponding to the mean calculated using the NISP method, are in 95% confidence interval of MCSM.

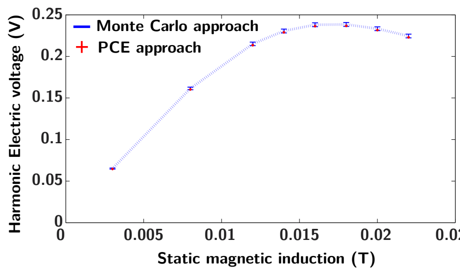

Fig. 2. Electric voltage versus magnetic flux density obtained by MCSM and NISP

We observe that the results obtained by the two different approaches are very close. Nevertheless, the computational time required for MCSM is about 4 times more than NISP method. Using NISP method, we can now estimate the first-order Sobol indices: S_1, S_2, S_3, S_4 corresponding respectively to μ, β, μ^* and λ^*. Fig. 3 plots the first-order Sobol indices versus different magnetic flux densities $\mathbf{B}_{dc}$.

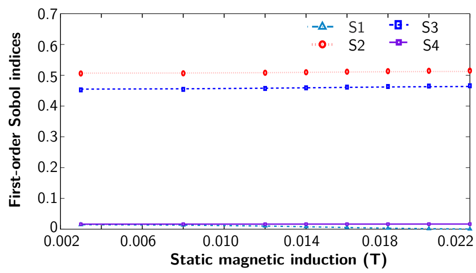

Fig. 3. First-order Sobol indices versus magnetic flux density

In Fig. 3, we can see that S_2 and S_3 are greater than S_1 and S_4. It means that the magnetostrictive coefficient β and the Lamé coefficient μ^* have a significant impact on the electric voltage v_{ac}. The permeability μ and the Lamé coefficient λ^* have almost no influence on the electric voltage v_{ac}. The Sobol indices are nearly constant and do not depend on $\mathbf{B}_{dc}$. The sum of the first-order Sobol indices is about 0.99 for all magnetic flux densities, the interaction effect of the inputs can be ignored. As a first conclusion, in order to limit the dispersion of the ME sensor characteristics, the variability of β and μ^* should be controlled during the fabrication process.

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