A Novel Evolution Strategy and its Application to Inverse Scattering in Microwave Imaging

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Abstract - An inversion methodology for microwave imaging **based** on an improved evolution strategy algorithm is proposed. **The nonparametric mutation process, adaptive trade-off between** exploration and exploitation, small population size and nonrevisiting property is well-suited for solving expensive evaluation **inverse problem. The numerical results as reported confirm positively** its feasibilities and advantages.

Index Terms—Inverse scattering, microwave imaging, **Evolution Strategy, numerical method.**

I. INTRODUCTION

Different inversion procedures have been developed for microwave imaging techniques by virtue of their exclusive merits and potentials in engineering applications. The most common and robust procedure is to recast the imaging problem into ^a minimization one, in which the second-order Born approximation (2BA) is used for scattered electric field computations [1].

The reversion is generally solved using ^a stochastic optimal algorithm [1]. However, the situations with both convergen^t speed and reversion accuracy of available approaches are still unsatisfactory. In this regard, ^a methodology based on improved evolution strategy (ES) algorithm is proposed.

To reduce the heavy computational burden of available stochastic optimal algorithms while guaranteeing their global search abilities, this paper puts forward two useful methods to improve the algorithm: 1) ^a completely derandomized selfadaptation scheme called covariance matrix adaptation (CMA) [2] is used. It can be improved by utilizing an evolution path rather than single search steps. The adaptation of strategy parameters in proposed algorithm takes place in the concep^t of mutative strategy parameter control (MSC). Strategy parameters are mutated and ^a new search point is generated by means of this mutated strategy parameter setting. 2) Proposes one solution to the revisit problem in the context of ES. We use an archive design using binary space partitioning (BSP) tree [3]. It not only is an efficient method to check for revisits and reduce the searching space and itself constitutes ^a novel adaptive mutation operator that has no parameter, which could perfectly overcome the shortcomings of CMA. $G(r, r)$
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Due to the inefficiency of Born-type approximations in scattered electric field computations, the frequency-domain finite element method is used to increase the accuracy of the field computations. To consider open boundaries, the secondorder absorbing boundary conditions are used.

II. MODELS AND METHODS

When ^a scatterer is illuminated by ^a single frequency *TM* incident wave, there will be conduction current or polarization current on its surface and in its body as shown in Fig.1.

Fig. 1. Geometric configuration of two-dimensional scattering case

Considering ^a nonmagnetic isotropic inhomogeneous lossy cylindrical scatterer in a finite space $V \subset R^3$, when the isotropic inhomogeneous lossy
nite space $V \subset R^3$, when the
inated in the free space by some
i, b, there will be an electric field
i. Under these assumptions, the
intering electric field (secondary scatterer is successively illuminated in the free space by some known magnetic waves (TM_z), there will be an electric field polarized in the *^z* direction. Under these assumptions, the polarized in the z
incident field $\vec{E}^i(\vec{r})$
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following Lippmann
 $\vec{E}(\vec{r}) = \vec{E}^i(\vec{r})$ $\vec{E}(\vec{r})$, the scattering electric field (secondary radiation) $\vec{E}'(\vec{r})$
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 E^{*i*} (\vec{r}) and the total field $\vec{E}(\vec{r})$ would satisfy the

following Lippmann-Schwinger integral equation
 $\vec{E}(\vec{r}) = \vec{E}^{\ell}(\vec$ following Lippmann-Schwinger integral equation

$$
\overrightarrow{E(r)} = \overrightarrow{E}(\overrightarrow{r}) + k_0^2 \int_{V} \overrightarrow{E(r)} [\overrightarrow{\varepsilon}, \overrightarrow{r}] - 1] G(\overrightarrow{r}, \overrightarrow{r}) dV
$$

+
$$
\int_{V} \nabla {\{\nabla \cdot \overrightarrow{E}(\overrightarrow{r}) \} [\overrightarrow{\varepsilon}, \overrightarrow{r}] - 1]} G(\overrightarrow{r}, \overrightarrow{r}) dV
$$

where, $\overrightarrow{r} \in R^3$ represents the observing point, $\overrightarrow{r} \in V$ is the

following Lippmann-Schwinger integral equation
 $\vec{E}(\vec{r}) = \vec{E}^T(\vec{r}) + k_0^2 \int_r \vec{E}(\vec{r}) [\vec{\epsilon}_r(\vec{r}) - 1] G(\vec{r}, \vec{r}) + \int_r \nabla {\nabla \cdot \vec{E}(\vec{r}) [\vec{\epsilon}_r(\vec{r}) - 1]} G(\vec{r}, \vec{r}) dV$

where, $\vec{r} \in R^3$ represents the observing poin $\vec{F} = R^3$ represents the observing point $\vec{F} = R^3$ represents the observing point $\vec{F} = R^3$ represents the observing point $\vec{F} = \vec{F}$, $(\vec{F}) = \varepsilon_r(\vec{F}) - \frac{\vec{F} \sigma(\vec{F})}{\omega \varepsilon_0}$ *Figure F <i>Figure F <i>Figure F Figure F <i>Figure F Figure F_{* *Figure *} $\int_{F} \nabla \{\nabla \cdot \vec{E}(\vec{r}) \} [\vec{\varepsilon}_{r}(\vec{r}) - 1] \} G(\vec{r}, \vec{r}) dV$
 $\in R^{3}$ represents the observing point,

region, $\overline{\varepsilon}_{r}(\vec{r}) = \varepsilon_{r}(\vec{r}) - \frac{j\sigma(\vec{r})}{\omega \varepsilon_{0}}$
 $\overline{\varepsilon}_{0}$ the permittivity of the surrounding Lippmann-schwinger integral e
 $\vec{E}(\vec{r}) = \vec{E}'(\vec{r}) + k_0^2 \int_V \vec{E}(\vec{r}) [\vec{\epsilon}_r(\vec{r}) -$
 $+ \int_V \nabla {\nabla \cdot \vec{E}(\vec{r}) [\vec{\epsilon}_r(\vec{r}) - 1]} G(\vec{r})$
 $\in R^3$ represents the observing

egion, $\vec{\epsilon}_r(\vec{r}) = \epsilon_r(\vec{r}) - \vec{r}$ + $\int_{V} \nabla {\overline{Y} \cdot \overline{E}(\overline{r'})} \left[\overline{\varepsilon_r}(\overline{r'}) - \overline{Y} \right] \cdot \overline{E}(\overline{Y}) \cdot \overline{E}(\overline{Y}) \cdot \overline{E}(\overline{Y})$

=, $\overline{r} \in R^3$ represents the obsection, $\overline{\varepsilon_r}(\overline{r'}) = \varepsilon_r(\overline{r'})$

sents the permittivity of the both following Lippmann-Schwinger integral equation

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 $\vec{E}(\vec{r}) = \vec{E}^T(\vec{r}) + k_0^2 \int_{\vec{r}} \vec{E}(\vec{r}) [\vec{\epsilon}_r(\vec{r}) - 1] G(\vec{r}, \vec{r})$

(ES)
 $+ \int_{\vec{r}} \nabla {\nabla \cdot} \vec{E}(\vec{r}) [\vec{\epsilon}_r(\vec{r}) - 1] {\nabla} (\vec{r}, \vec{r}) dV$

where, $\vec{r$ $(\vec{r}) = E(r) + k_0^2 \int_r E(r) [\varepsilon_r(r) - 1] G(r, \vec{r})$
+ $\int_r \nabla {\nabla \cdot \vec{E}(\vec{r})} [\tilde{\varepsilon_r}(\vec{r}) - 1] G(r, \vec{r})$
 R^3 represents the observing po
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th $G(\vec{r}, \vec{r})$ is the free space source region, $\overrightarrow{\varepsilon_r(r)} = \varepsilon_r(\overrightarrow{r})$
represents the permittivity of the region, $\overline{\varepsilon_r}(\overrightarrow{r}) = \varepsilon_r(\overrightarrow{r}) - \frac{j\sigma(\overrightarrow{r})}{\omega \varepsilon_0}$
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) is the free space Green's function and $\overrightarrow{r}, \overrightarrow{r}$ = $e^{-jk_0|\overrightarrow{r}-\overrightarrow{r}|}/4\pi |\overrightarrow{r}-\overrightarrow{r}|$ on, $\vec{\epsilon}_r(\vec{r}) = \epsilon_r(\vec{r})$

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the free space Green's
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ding background
and is given by represents the permittivity of the surrounding background, (r, r') is the free space Green's function and is given by

$$
G(\vec{r},\vec{r})=e^{-jk_0|\vec{r}-\vec{r}|}/4\pi|\vec{r}-\vec{r}|
$$
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of $G(\vec{r}, \vec{r'}) = e^{-\frac{\beta}{\sqrt{2}}\vec{r}}/4\pi |\vec{r} - \vec{r'}|$

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We this approximation, one obtains the For the space Green's
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two-dimensional (2D) prough. As a result, the total

ated by the second-order Environmental problems that For ^a two-dimensional (2D) problem, if the scatterer is \mathcal{P} weak enough. As a result, the total electric field can be approximated by the second-order Born approximation. Using this approximation, one obtains the scattering field as

$$
E^{S}(\vec{r}) \approx E_{\text{Born}}^{(2)}(\vec{r}) = \int_{V} [\vec{E}'(\vec{r}) + E_{\text{Born}}(\vec{r})] \pi(\vec{r}) G(\vec{r}, \vec{r}) dV \tag{3}
$$

this approximation, one obtains the scattering field as
 $E^S(\vec{r}) \approx E_{\text{down}}^{(2)}(\vec{r}) = \int_{\vec{r}} [\vec{E}^T(\vec{r}) + E_{\text{down}}(\vec{r})] x(\vec{r}) G(\vec{r}, \vec{r}) dV$

Consider a uniform cylindrical object with a va

dielectric permittivity $\varepsilon(x, y$ Consider a uniform cylindrical object with a variable Consider a uniform cylindrical object with a variable
dielectric permittivity $\varepsilon(x, y)$, the discrete counterpart of (3)
can be obtained as
 $E_{scatt(2BA)}^v = \sum_{i=1}^N G_{m,i}^v \tau_i \cdot [E_{inc,i}^v + \sum_{n=1}^N G_{i,n}^v \tau_n \cdot E_{inc,i}^v]$ (4)
 $G_{$ can be obtained as

$$
F_{scatt(2BA)}^{\nu} = \sum_{l=1}^{N} G_{m,l}^{\nu} \tau_{l} \cdot [E_{inc,l}^{\nu} + \sum_{n=1}^{N} G_{l,n}^{\nu} \tau_{n} E_{inc,l}^{\nu}]
$$
(4)

$$
G_{m,l}^{\nu} = \frac{-jk_0^2}{4} \int_{S} H_0^{(2)} (k_0 \rho(x_m^{\nu}, y_m^{\nu}, x^{\nu}, y^{\nu})) dx^{\nu} dy^{\nu}
$$
(5)

can be obtained as
\n
$$
E_{scatt(2BA)}^v = \sum_{l=1}^N G_{m,l}^v \tau_{l} \cdot [E_{inc,l}^v + \sum_{n=1}^N G_{l,n}^v \tau_n E_{inc,l}^v]
$$
\n
$$
G_{m,l}^v = \frac{-jk_0^2}{4} \int_{S_l} H_0^{(2)} (k_0 \rho(x_m^v, y_m^v, x^v, y^v)) dx^v dy^v
$$
\n
$$
(m = 1, \dots, M; l = 1, \dots, N; v = 1, \dots, V)
$$
\nwhere, $\nu(V)$ is the incident angle, $m(M)$ is the number of

 $G_{m,l}^{\nu} = \frac{-jk_0^2}{4} \int_{S} H_0^{(2)}(k_0 \rho(x_m^{\nu}, y_m^{\nu}, x^{\nu}, y^{\nu})) dx^{\nu} dy^{\nu}$
 $(m=1,\dots,M; l=1,\dots,N; \nu=1,\dots,N)$

(*V*) is the incident angle, $m(M)$ is the num

ment points, τ_{ρ} is the unknown quantity of

ain given by $\tau = \{ \varepsilon$ (*m* = 1, ..., *M*; *l* = 1, ..., *N*; *y* = 1, ..., *V*)

(*m* = 1, ..., *M*; *l* = 1, ..., *N*; *y* = 1, ..., *V*)

(re, *v*(*V*) is the incident angle, *m*(*M*) is the

surement points, τ_p is the unknown quantitive
 $G_{m,l}^{\nu} = \frac{-jk_0^2}{4} \int_{S} H_0^{(2)}(k_0 \rho(x_m^{\nu}, y_m^{\nu}, x^{\nu}, y^{\nu})) dx^{\nu} dy$
 $(m = 1, \dots, M; l = 1, \dots, N; \nu = 1, \dots, V)$

re, $\nu(V)$ is the incident angle, $m(M)$ is the nur

surement points, τ_{ρ} is the unknown quantity of

domain gi *m*_{*i*} *A* \rightarrow *M A i M i C M i M j M i W (M)* is the incident angle, *m*(*M)* is the nent points, τ_p is the unknown quantiting given by $\tau = [\varepsilon(x, y)/\varepsilon_0$ -1] and is taken measurement points, τ_p is the unknown quantity of the p^{th} sub-domain given by $\tau = [\varepsilon(x, y)/\varepsilon_0 - 1]$ and is taken as constant
sub-domain given by $\tau = [\varepsilon(x, y)/\varepsilon_0 - 1]$ and is taken as constant

in an element, k_0 is the wavenumber in the external medium,

 ρ is given by is given by $\rho(x_m^y, y_m^y, x^y, y^z) = \sqrt{(x_m^y - x^2)^2 + (y_m^y - y^2)^2}$,
zeroth order Hankel functions of the second
ment number of the mesh.
Under such assumptions, the inverse scatt
be recast into a discretized form as $L^{(2)}$ $\frac{1}{2} \left(v_m^y - y^2 \right)^2$, $H_0^{(2)}(k_0 \rho)$ is
the second kind, N is the
erse scattering problem the zeroth order Hankel functions of the second kind, N is the element number of the mesh.

Under such assumptions, the inverse scattering problem can be recast into ^a discretized form as

$$
\Im = \sum_{\nu=1}^{V} \sum_{m=1}^{M} \left| E_{scat,m}^{v} - \sum_{l=1}^{N} G_{m,l}^{v} \tau_{l} \cdot [E_{inc,l}^{v} + \sum_{n=1}^{N} G_{l,n}^{v} \tau_{n} \cdot E_{inc,l}^{v}] \right|^{2} \quad (6)
$$

$$
E_{inc}^{v}(x, y) = \exp \{ \int [x \cos v_{inc}^{v} + y \sin v_{inc}^{v}] \}, v = 1, ..., V \quad (7)
$$

where the incident angle v_{inc}^{v} is given by

$$
v_{inc}^{v} = (v-1) \frac{2\pi}{V} (v=1, ..., V) \quad (8)
$$

$$
E_{inc}^{v}(x, y) = \exp\{f[x\cos v_{inc}^{v} + y\sin v_{inc}^{v}]\}, v = 1, ..., V \quad (7)
$$

where the incident angle
$$
v_{inc}^v
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 is given by
\n
$$
E_{inc}^v(x, y) = \exp\{f[x\cos v_{inc}^v + y\sin v_{inc}^v]\}, v = 1, ..., V
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\n
$$
v_{inc}^v = (v-1)\frac{2\pi}{V}(v=1, ..., V)
$$
\n(8)
\nThe polar coordinates of the *m*th measurement point are

(1) (1,...,) *inc ^v ^v ^V* $v_{inc}^{\nu} = (\nu - 1) \frac{2\pi}{V} (\nu = 1, ..., V)$ (8)

r polar coordinates of the *mth* measurement point are

i, and *r* is set to a fixed value and v_m^{ν} is given by
 $v_m^{\nu} = \frac{\partial + v_{inc}^{\nu} + 2(m-1)(\pi - \partial)/(M-1)}{m-1 - M}$ (9) The polar coordinates of the *^mth* measurement point are The (r, v_m^{ν})
where To (r, v_m^{ν}) , and *r* is set to a fixed value and v_m^{ν} is given by

$$
v_m^{\nu} = \partial + v_{inc}^{\nu} + 2(m-1)(\pi - \partial)/(M-1)
$$

\n
$$
m = 1,...,M; v = 1,...,V
$$
\n(9)

where ∂ is a fixed angle.

 $(\partial + v_{inc}^{\nu} + 2(m-1)(\pi - \partial)/({M-1})$
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small constant predefined by $D_m = C + D_{inc} + 2(m - m)$
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is a fixed angle.

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(τ_{min} is small constant $v_m^v = \partial + v_{inc}^v + 2(m-1)(\pi - \partial)/(M - \partial)$
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ixed angle.
 v_m^v the computational burden of

one will always test if

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 $m = 1,..., M; v = 1,..., V$
 ∂ is a fixed angle.

reduce the computational

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ed at the beginning of 6 $v_m^v = \frac{\partial + v_m^v}{\partial x} + 2(m-1)(\pi - \frac{\partial}{\partial x})/(M - m) = 1,..., M; v = 1,..., V$
 $\frac{\partial}{\partial y}$ is a fixed angle.

reduce the computational burden of dology, one will always test if the *c*, is small constant predefined by $m = 0 + U_{inc} + 2$
 $v = 1,..., M; v =$
 $v = 1,..., M; v =$
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 $v = 1$
 To reduce the computational burden of the proposed methodology, one will always test if the condition methodology, one will always test if the condition $\tau \ge \tau_{min}$ (τ_{min} is small constant predefined by the user) is satisfied at the beginning of each iteration step. The computation producer for the term $G \cdot \tau \cdot E$ wil satisfied at the beginning of each iteration step. The satisfied at the beginning of each iteration step. The computation producer for the term $G \cdot \tau \cdot E$ will be activated only when this condition is satisfied. Such approach is particularly computational efficient when the s only when this condition is satisfied. Such approach is particularly computational efficient when the scatterers are taking up only ^a few fraction of the whole solution domain.

III. NUMERICAL VALIDATIONS

The first application is ^a single weak scatter. The scatter is a homogeneous cylinder of radius $r = 0.4 \lambda_0$ with $\tau = 1.2$. The center of the circular is at $(x_0, y_0) = (-0.137 \lambda_0, -0.137 \lambda_0)$. The solution domain of an $a \times a$ ($a = \lambda_0$) square is partitioned into $N=20 \times 20$ sub-domains. 32 observing points are distributed uniformly on a circumference of radius $b = 0.84 \lambda_0$, and *V* is set to be 1. Fig. 2 compares the model and the inversed scatting electric fields using the 2BA (2BA) based procedure and the proposed (Proposed) procedure after 50 iterations. The inversed τ is 1.1382 using the 2BA, which is compared to a value of $\tau = 1.22$ using the Proposed. Comparisons of the reversed scattering electric fields at the measurement or observation points using the two techniques are given in Fig. 3. Obviously, the Proposed outperforms 2BA in both efficiency and accuracy.

Fig. 2. The original models (a,c) and the reversed scatting electric fields (b,d) using the 2BA based (a,b) and the Proposed (c,d) procedures.

Fig. 3. The actual and the reversed scattering electric fields at the measured points using the 2BA based and the Proposed procedures.

The second application is ^a multiple strong scatter case. Four homogeneous strong cylindrical scatters of radius $r=0.1 \lambda_0$ are centered, respectively, at $(-0.11 \lambda_0, -0.11 \lambda_0)$, $(0.11 \lambda_0, -0.11 \lambda_0), (0.11 \lambda_0, 0.11 \lambda_0), (-0.11 \lambda_0, 0.11 \lambda_0)$ with dielectric permittivities of $\varepsilon_1=3.5$, $\varepsilon_2=2.0$, $\varepsilon_3=3.0$, and ϵ ₄=2.0. 100 observing points are distributed uniformly on a circumference of radius $b=0.84 \lambda_0$, and $V=1$. Also, the Proposed and 2BA are employed to inverse the scatters. It is found that it is difficult to obtain accurate results of the scatter parameters using 2BA while sufficiently accurate results of $\varepsilon_1 = 3.48$, $\varepsilon_2 = 2.14$, $\varepsilon_3 = 2.79$, $\varepsilon_4 = 1.98$ are attained by the Proposed after only 50 iterations. The reversed scattered electric fields on the observing points using the two procedures are compared in Fig. 4. Clearly, the Proposed will reverse ^a complex nonlinear inverse scattering problem with good accuracies. However, the reliable results of the same problem cannot be realized using available 2BA.

Fig. 4. Comparison of reversed scattered electric fields using different procedures.

IV. REFERENCES

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