Design Optimization of a Loudspeaker Utilizing Sampling-based Sensitivity Information of a Hyper-spherical Local Window

Nak-Sun Choi[1], Dong-Wook Kim[1], K. K. Choi[2], and Dong-Hun Kim[1]
[1]Department of Electrical Engineering, Kyungpook National Univ., Daegu 702-701, Republic of Korea
[2]Mech. and Ind. Eng., Univ. of Iowa, Iowa City, USA and Naval Arch. & Ocean Eng., Seoul Nat'l Univ., Korea
dh29kim@ee.knu.ac.kr

***Abstract*—This paper proposes an efficient optimization method which can deal with electromagnetic design problems with high-dimensional design variables. To achieve this, accurate surrogate models for performance functions of interest are generated at each intermediate design point based on the universal Kriging method combined with a new-type local window, called the hyper-sphere, and the truncated Gaussian sampling. Due to exploiting the first-order design sensitivity values extracted from the surrogate models, the optimal design can be obtained even with relatively small iterative designs. The proposed method is applied to a loudspeaker design with 12 design variables and then its efficiency is thoroughly investigated by comparing with existing methods.**

***Index Terms*— Electromagnetics, metamodeling, optimization, sensitivity analysis.**

I. Introduction

When dealing with real design problems of electromagnetic (EM) appliances, engineers often encounter high-dimensional (H-D) optimization problems which usually have more than 10 design variables. In the case, they can't easily make a decision on which optimization method is efficient from the viewpoint of computational cost and solution accuracy. In the literature, most of the published research works belongs to low-dimensional design problems and deterministic design optimization methods, consisting of either sampling-based or sensitivity-based approaches, have advantages and dis-advantages [1]. To deal with the H-D design problems, authors have proposed a hybrid method where the universal Kriging model was combined with a hyper-cubic local window concept and Latin Centroidal Voronoi Tessellations (LCVT) sampling [2]. However the method was applied only to low dimensional 2-D and 4-D design problems. Thus, it is still desirable to develop a new local window type and an efficient sampling strategy, which is more adequate to extract design sensitivity values with high precision from the surrogate models of H-D problems. To overcome the aforementioned weaknesses, a hyper-spherical local window and a truncated Gaussian sampling (TGS) are introduced to the sampling-based sensitivity method. Then the proposed method is applied to 12-D design problem of a loudspeaker and its efficiency is examined compared with highly-tuned commercialized optimizers.

II. Sampling-based Sensitivity Method

While the existing Kriging methods use a global window and high-order polynomials, the proposed method utilizes surrogate models based on a local window and the first-order basis function. Exploiting the derivatives of surrogate models generated in a local design window at the center of a current design point, the improved design is obtained by means of sensitivity-based searching techniques.

A. Derivative of a predicted Kriging model

In the Kriging method, the outcomes are considered as a realization of a stochastic process. The goal is to estimate a response $\mathbf{y}=[y(\mathbf{x}_1),\cdots,y(\mathbf{x}_n)]^T$ with $y(\mathbf{x}_i)\in\mathbf{R}^1$ based on n sample points, $\mathbf{x}=[\mathbf{x}_1,\cdots,\mathbf{x}_n]^T$ with $\mathbf{x}_i\in\mathbf{R}^m$. The response consists of a summation of two parts: mean structure of the response $\mathbf{F\beta}$ and realization of the stochastic process $\mathbf{e}$ as

$$\mathbf{y}=\mathbf{F\beta}+\mathbf{e} \tag{1}$$

where $\boldsymbol{\beta}$ is the vector of regression coefficient. Applying fairly routine mathematical processes such as the maximum likelihood estimator and the Lagrange multiplier, the prediction $\hat{y}$ of (1) which interpolates the n sample points around the point $\mathbf{x}_0$ is expressed as

$$\hat{y}(\mathbf{x}_0)=\mathbf{f}_0^T\boldsymbol{\beta}+\mathbf{r}_0^T\mathbf{R}^{-1}(\mathbf{y}-\mathbf{F\beta}) \tag{2}$$

where $\mathbf{f}_0$ is the basis function vector at $\mathbf{x}_0$, $\mathbf{r}_0$ is the correlation vector between $\mathbf{x}_0$ and samples $\mathbf{x}$, and $\mathbf{R}$ is the symmetric correlation matrix. Finally, the derivative $\hat{y}'$ of the prediction model (2) at $\mathbf{x}_0$ is obtained as [3]

$$\hat{y}'(\mathbf{x}_0)=\mathbf{J}_\mathbf{f}^T\boldsymbol{\beta}+\mathbf{J}_r^T\mathbf{r}_0 \tag{3}$$

where $\mathbf{J}_\mathbf{f}^T$ and $\mathbf{J}_r^T$ denote the Jacobian transformations of $\mathbf{f}_0$ and $\mathbf{r}_0$, respectively.

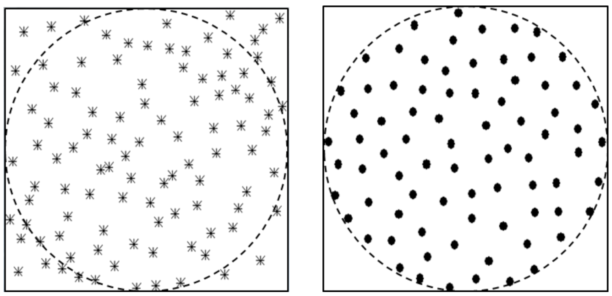
B. Hyper-spherical local window and sampling strategy

To efficiently carry out the sampling-based sensitivity analyses for H-D optimization problems, a hyper-spherical local window and TGS technique are newly adopted. Even though the hyper-cubic window combined with the LCVT sampling has been widely used for Kriging models of electromagnetic problems, it is not efficient and suitable for solving H-D design optimization problems. That is because the sampling points of the existing method are liable to be located around the corners of the hyper-cube as the dimension of design variables increases. It is well known that there are extremely few samples inside the hyper-sphere inscribed in the hyper-cube when the dimension is 10 or higher [3].

To overcome the aforementioned problem and obtain more accurate design sensitivity information, a hyper-spherical local window is combined with TGS technique. The radius R of the hyper-sphere is determined in *nd* dimensional hyper-spherical coordinates as:

$$R = cR^{nd} \tag{4}$$

where c is the coefficient which is usually between 2~5%, and R^{nd} is the unit radius in *nd* dimensions. The basic idea of TGS is that a random vector in hyper-spherical coordinates is first generated and then this random vector is transformed onto a random vector in rectangular coordinates. Therefore the method can efficiently produce more uniform random samples in the hyper-sphere. For instance, the comparison of the two sampling techniques, LCVT and TGS, in two dimensions is presented in Fig. 1 where the goal is to generate 79 samples inside the hyper-sphere inscribed in the hyper-cube.



(a) LCVT (100 samples in hyper-cube) (b) TGS (79 samples)

Fig. 1. Comparison between two sampling strategies.

III. Results

The loudspeaker design problem in [4] is considered to demonstrate the effectiveness of the proposed method for EM device designs. Fig. 2 shows the configuration of the loudspeaker with 12 design variables. The objective function f is defined to minimize the loudspeaker mass M and the constraint function g is set to keep the average flux density of the air gap being more than B_0=1.8 T, as

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{d}) = \mathrm{M}(\mathbf{d}) \\ &\text{subject to} \quad g(\mathbf{d}) = \mathrm{B}_0 - \mathrm{B}(\mathbf{d}) \geq 0 \end{aligned} \tag{5}$$

where d is the design variable vector.

To take into account the nonlinear property of the steel yoke, the prediction of the objective and the constraint functions was made by a commercial analysis tool, called MagNet VII [5]. For investigating the efficiency of the proposed method, four different optimization methods were tested: 1). evolutionary-based stochastic algorithm embedded in OptiNet [5], 2). FDM-based sensitivity optimization provided by Matlab, 3). sampling-based sensitivity optimization with the hyper-cubic local window and LCVT, and 4). proposed sampling-based sensitivity optimization with the hyper-spherical local window and TGS. The two sampling-based sensitivity optimization methods (3) and (4) were carried out with 25 initial samples. The sequential quadratic programing (SQP) technique was used for all of the three sensitivity-based methods (2), (3) and (4). Starting with the same initial design, the obtained optima are presented in Table I where the four optimum points are quite different to each other. From the results, it is inferred that the design problem itself has a number of local minima, and it is observed that the proposed method yields the best optimum solution in terms of the two performance values (f and g). Fig. 3 shows the performance indicator of the four different optimization methods. Among them, the proposed method requires the lowest number of iterative designs for obtaining an optimum. It implies that the proposed method produces more accurate sensitivity value than the other sensitivity-based optimization methods (2) and (3). Meanwhile, the numbers of finite element analysis (FEA) calls between the FDM-based and the proposed methods (2) and (4) show a slight difference.

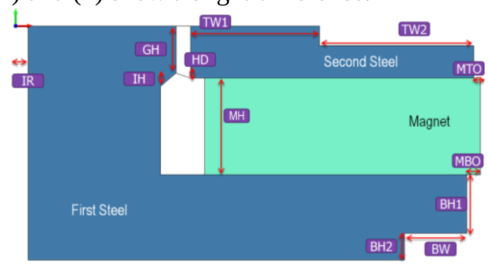


Fig. 2. Two-dimensional axisymmetric configuration of a loudspeaker.

TABLE I
OPTIMA OBTAINED FROM FOUR DIFFERENT OPTIMIZATION METHODS

Design variables	Initial design	Evolution -based	FDM-based	Hyper-cube LCVT	Hyper-sphere TGS
BH1	9.63	2.82	7.47	5.18	5.18
BH2	6.15	4.23	7.67	5.03	5.59
BW	12.12	12.09	12.70	11.96	12.27
HD	2.64	0.15	0.10	0.48	0.18
IH	4.80	1.52	1.93	1.60	1.37
IR	4.98	2.11	3.96	4.04	3.76
MBO	1.50	1.45	4.06	3.53	4.04
MH	15.75	13.80	12.88	9.58	7.92
MTO	2.97	1.17	3.18	0.79	1.30
TH	3.10	3.63	2.08	4.01	3.28
TW1	22.12	19.51	11.02	13.90	14.66
TW2	19.28	4.85	8.69	11.38	11.46
f (kg)	3.57	1.51	1.16	1.02	1.01
g (T)	1.73	1.80	1.80	1.80	1.80

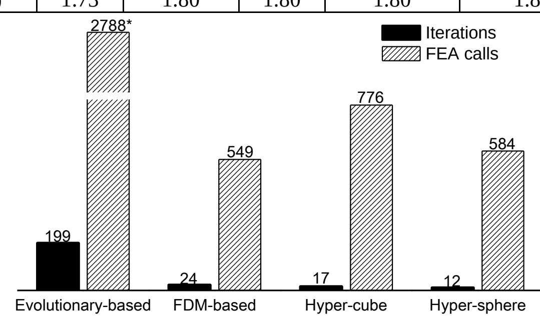


Fig. 3. Performance indicator of four different optimization methods: the number, 2778*, is estimated.

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