Fast Shape Optimization of Microwave Devices Based on Parametric Reduced Order Models

Stefan Burgard, Ortwin Farle, Philipp Loew, and Romanus Dyczij-Edlinger
Department of Physics and Mechatronics, Saarland University,
Building C6 3, University Campus, D-66123 Saarbrücken, Germany
edlinger@lte.uni-saarland.de

***Abstract*—Numerical simulation methods at the fields level, such as the finite-element method, are highly accurate but computationally expensive. In the context of mathematical optimization, this implies that the cost function, which needs to be evaluated a large number of times, is very expensive to compute. To overcome this shortcoming, the present paper proposes to employ parametric reduced-order models for computing the cost function: They introduce low systematic error, require little memory, and allow for hundreds of model evaluations per second. The great utility of the suggested approach in both deterministic and stochastic optimization methods is demonstrated by a numerical example, featuring 11 geometric parameters.**

***Index Terms*—Computer aided engineering, design optimization, numerical models, reduced order systems.**

I. Introduction

Since fields-level simulations, specifically finite-element (FE) computations, are often considered as too expensive for calculating the cost function at every optimization step, one often resorts to cheaper surrogate models. Common approaches include space-mapping [1], response surface modeling [2], and Kriging [3]. As a highly accurate and efficient alternative for linear time-invariant systems, the authors of this paper propose to employ parametric reduced-order models (PROMs) for computing the cost function.

Parametric model-order reduction (PMOR) involves a two-stage process: The first and computationally more expensive step is PROM generation. It needs to be performed only once, before the optimization process begins. The second step consists of evaluating the PROM for an arbitrary parameter vector. It is very cheap but needs to be performed once or even multiple times at every optimization step, to compute the cost function. In the numerical example below, PROM generation and evaluation times are 8840 s and $9.5 \cdot 10^{-4}$ s, respectively.

PROMs offer the following characteristic features:

- The PROM is constructed from the fields-level model in a systematic way that lends itself well to automation.
- It maintains the structure and parameter-dependence of the fields-level model and thus preserves important system properties.
- The systematic error of the PMOR process is easy to control. If necessary, it can be made of the same order as that of the fields-level model.
- PROM dimension depends on the complexity of the system response and the error threshold chosen, respectively, but not on the dimension of the fields-level model.

II. Parameter-dependent Finite Element Model

We consider an N dimensional time-harmonic electromagnetic FE system $\Sigma(f, \mathbf{p})$ with Q dimensional input and output vectors $\mathbf{u}, \mathbf{y} \in \mathbb{C}^Q$, which depends on the frequency $f \in \mathbb{R}$ and a vector $\mathbf{p} \in \mathbb{R}^P$ of P design parameters. Its general form is:

$$\Big(\sum_{i=1}^{I} \phi_i(f)\mathbf{A}_i(\mathbf{p})\Big)\mathbf{x}(f,\mathbf{p}) = \Big(\sum_{j=1}^{J} \theta_j(f)\mathbf{B}_j\Big)\mathbf{u}, \tag{1a}$$

$$\mathbf{y}(f,\mathbf{p}) = \Big(\sum_{j=1}^{J} \eta_j(f)\mathbf{B}_j^T\Big)\mathbf{x}(f,\mathbf{p}), \tag{1b}$$

wherein $\mathbf{x} \in \mathbb{C}^N$ is the generalized state, $\mathbf{A}_i \in \mathbb{R}^{N\times N}$ and $\mathbf{B}_j \in \mathbb{R}^{N\times Q}$ are coefficient matrices, and $\phi_i, \theta_j, \eta_j : \mathbb{R} \to \mathbb{C}$ are continuous functions. The cost function $c(f,\mathbf{p}) : \mathbb{R}^{P+1} \to \mathbb{R}$, which is computed via $\mathbf{y}(f,\mathbf{p})$, is assumed to be smooth enough to allow for gradient-based optimization.

III. Parametric reduced order model

By applying the PMOR method of [4] to the FE system (1), one obtains a structurally identical PROM $\widetilde{\Sigma}(f,\mathbf{p})$, which reads

$$\Big(\sum_{i=1}^{I} \phi_i(f)\widetilde{\mathbf{A}}_i(\mathbf{p})\Big)\widetilde{\mathbf{x}}(f,\mathbf{p}) = \Big(\sum_{j=1}^{J} \theta_j(f)\widetilde{\mathbf{B}}_j(\mathbf{p})\Big)\mathbf{u}, \tag{2a}$$

$$\mathbf{y}'(f,\mathbf{p}) = \Big(\sum_{j=1}^{J} \eta_j(f)\widetilde{\mathbf{B}}_j^T(\mathbf{p})\Big)\widetilde{\mathbf{x}}(f,\mathbf{p}), \tag{2b}$$

with $\widetilde{\mathbf{A}}_i \in \mathbb{R}^{n\times n}$, $\widetilde{\mathbf{B}}_j \in \mathbb{R}^{n\times Q}$, $\widetilde{\mathbf{x}} \in \mathbb{C}^n$, and $\mathbf{y}' \in \mathbb{C}^Q$. The key feature of $\widetilde{\Sigma}(f,\mathbf{p})$ is that its outputs are highly accurate over the considered parameter domain, $\mathbf{y}'(f,\mathbf{p}) \approx \mathbf{y}(f,\mathbf{p})$, even though the PROM dimension is very small, $n \ll N$. Thus, the cost function $c(f,\mathbf{p})$ can be approximated very efficiently, using $\mathbf{y}'$.

IV. Numerical Example: Waveguide Filter Optimization

Fig. 1 gives a schematic representation of a bandpass filter after [5]. It consists of seven resonant irises separated by sections of empty waveguide (WG); see inset of Fig. 3 for dimensions. The design parameters are given by the widths w_i and heights h_i of the resonant windows as well as the lengths of the WG sections D_i. Assuming that the geometry is symmetric, the total number of parameters is $P = 11$.

A. Cost Function

The nominal filter response is that of a bandpass of maximally flat amplitude (Butterworth), with center frequency f_c = 12.55 GHz and lower half-power frequency f_0 = 11.5 GHz. The frequency band of interest is $8\ldots16$ GHz [5]. The

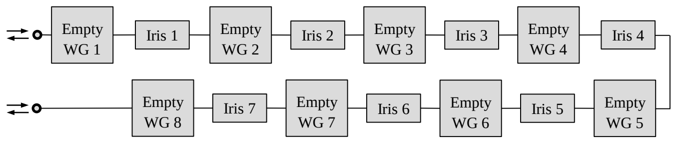


Figure 1: WG bandpass filter with seven irises.

Table I: Computational Data*.

Model	PROM	FE model
Dimension	70	124,370
Model generation (s)	8839.72	-
Model evaluation (s)	$9.47 \cdot 10^{-4}$	23.61
Avr. error in $\mathbf{S}$	$3.95 \cdot 10^{-4}$	-
Cost function evaluation (s)	1.01	18,982.69

* Single thread performance on Intel Xeon E5620.

analytical reference curve $|S^{\text{ref}}_{1,2}(f, f_0, f_c)|$ is calculated from a simple network model, neglecting dispersion and parasitic coupling. The cost function is taken to be the least-squares fit of the actual filter response, sampled at $N_f = 201$ equidistant frequency points:

$$c(\mathbf{p}) = \frac{1}{N_f} \sum_{n=1}^{N_f} \left(|S^{\text{ref}}_{1,2}(f_n, f_0, f_c)| - |S_{1,2}(f_n, \mathbf{p})| \right)^2 . \quad (3)$$

B. Modeling

First, a PROM $\widetilde{\Sigma}(f, w, h)$ is generated from the FE analysis of a single parameter-dependent iris for the first 5 dominant modes, utilizing symmetry. Expected PROM error is in the order of $10^{-3} \ldots 10^{-2}$. Second, standard WG network techniques are employed to interconnect the irises according to Fig. 1. Table I shows that standard FE analysis would take almost 19000 s to calculate a single cost function based on 203 frequency points, whereas the proposed method just requires 1 s. Thus, the time for constructing the PROM is well invested.

C. Optimization Results

As a representative for a deterministic method, we have applied a quasi-Newton active set method [6], using a crude hand-calculation for the starting point. The gradient of the cost function was approximated by finite-differences, requiring another 22 cost function evaluations per iteration. The method terminated successfully after 46 steps.

To demonstrate stochastic optimization, the genetic algorithm [7] from the optimization toolbox of MATLAB was used. The population size was 1600, the number of elites 20, and the cross-over and mutation rates were 80% and 20%, respectively. Convergence was obtained after 495 generations.

The convergence criterion was $c^{\text{tol}} = 6 \cdot 10^{-6}$. Fig. 2 presents the convergence behavior of both approaches. Fig. 3 shows that both methods yield very similar results, close to the reference curve. Fig. 4 compares the PROM results for the optimized configurations to FE calculations: The largest pointwise error is 10^{-2}, in good accordance with the accuracy of the PROM.

V. Conclusions

PMOR provides a powerful tool for both deterministic and stochastic optimization at the fields level. Even though the prototype methods of this abstract achieve speed-ups by orders of magnitudes compared to conventional FE analysis, they yet leave ample room for improvement. In the presentation, we shall give optimized methods, including parallelization.

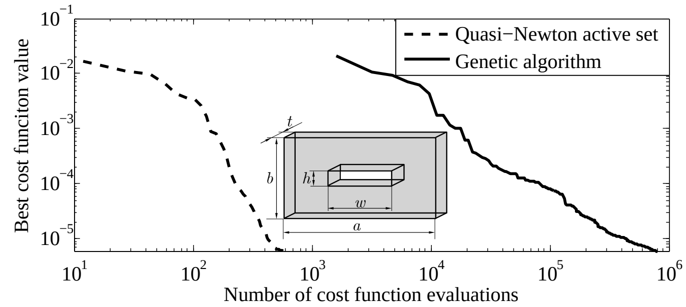
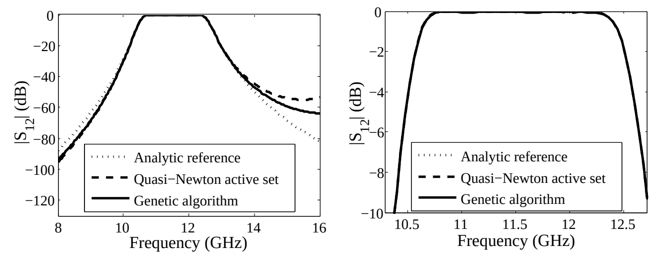


Figure 2: Convergence of optimization algorithms. Inset shows iris. Dimensions: t = 1 mm a = 22.86 mm, b = 10.16 mm.



(a) Entire frequency band.

(b) Close-up view on passband.

Figure 3: Amplitude response of optimized bandpass filter.

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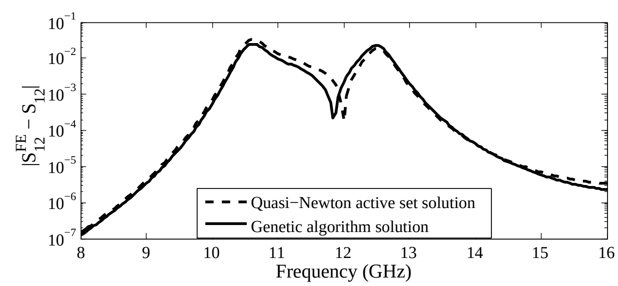

Figure 4: Error of PROM compared to FE calculations.