Benefits of waveform relaxation method and space mapping for the optimization of multirate system

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***Abstract*—For the optimization of a component from a multirate system, the article presents benefits of the joint use of several models for the optimization and of fixed-point strategy for the modeling. Thus, space mapping allows reducing time computation of an optimization process by limiting the number of evaluations of time consuming models. In the case of multirate problems, these models can be modeled by Waveform Relaxation Method to provide an additional time saving.**

***Index Terms*—Differential algebraic equations, Optimization methods**

I. Introduction

In the framework of the optimization of a multi-physic system, it is necessary to perform a coupling of different numerical models which be consistent. But modeling of a system including components with very different time constants is particularly problematic. On one hand, a strong coupling involves a time discretization according to the smallest time constant, and thus a long computation time. On the other hand, a weak coupling implies hypothesis which involve a lack of consistency of the results.

The Waveform Relaxation Method (WRM) [1],[2] is an iterative process which converges in theory to the solution of a strong coupling, but models each component with respect to its own time constant, leading to a save of time. Moreover, an optimization process involves a huge number of model evaluations. If the model is time consuming the optimization can be extremely long to execute. With the aim to reduce even more the optimization time, a Space Mapping (SM) strategy [3],[4],[5] is set up. This is an iterative process which requires at least two models of the same device, but with different accuracy and computing time. The fastest one is used during optimization; the most time consuming is evaluate one time per iteration to correct the other model. Typically, the models used for a space mapping are lumped-mass model and finite element model.

The first two parts of the article present the waveform relaxation method and the output space mapping technique. In the last part, these methods are applied to the minimization of a transformer mass.

II. Waveform relaxation method

Let a Differential Algebraic Equation (DAE) representing a system on the time domain $T = [t_0, t_f]$:

$$\dot{y}(t) = h(y(t), z(t)), \tag{1}$$

$$0 = g\left(y(t), z(t)\right). \tag{2}$$

The system is decomposed into r subsystems, each subsystem i satisfying:

$$\dot{y}_i(t) = h_i(y(t), z(t)), \tag{3}$$

$$0 = g_i(y(t), z(t)). \tag{4}$$

The WRM produces iteratively an approximation $(\tilde{y}^k(t), \tilde{z}^k(t))$ of the solution $(y(t), z(t))$, where k is the iteration index. The initial iterate is fixed using the known value of y and z in t_0: $\tilde{y}^0(t) = y(t_0),\ \tilde{z}^0(t) = z(t_0),\ \forall t \in T$. Then the subsystems are solved successively from the subsystem 1 to r. At the k-iteration, subsystem i is solved, using variables from subsystems 1 to $(i-1)$ at iteration k and variables from subsystems $(i+1)$ to r at iteration $(k-1)$ as source terms.

Each subsystem is solved using its own time discretization. This strategy brings a gain of time computation if the most time consuming subsystems have a low dynamic.

The algorithm stops when the norm of the difference between two successive iterates is less than a given tolerance.

III. Output space mapping

The following optimization problem has to be solved:

$$x_f^\star = \arg\min_{x_f} \|f(x_f) - y\| \text{ such that } k_f(x_f) \leq 0. \tag{5}$$

Objective function f and constraints k_f form the fine model, with both high precision and time computation. A second model of the same phenomena is considered: c and k_c, the coarse model, faster but less accurate. The optimization problem associated is:

$$x_c^\star = \arg\min_{x_c} \|c(x_c) - y\| \text{ such that } k_c(x_c) \leq 0. \tag{6}$$

The principle of the Output Space Mapping (OSM) is to solve a corrected coarse problem per iteration, then to evaluate the fine model to obtain new correctors of the coarse model

$$
\begin{array}{l}
j = 0 \\
O^j = I_m,\ \tilde{O}^j = I_p \\
\text{do} \\
\qquad x^j = \arg\min_{x_c} \|O^j.c(x_c) - y\| \text{ s. t. } \tilde{O}^j.k_c(x_c) \le 0 \\
\qquad O^{j+1}_{i,i} = \dfrac{f_i(x^j)}{c_i(x^j)},\ \tilde{O}^{j+1}_{l,l} = \dfrac{k_{f,l}(x^j)}{k_{c,l}(x^j)},\ \forall i, l \\
\qquad j = j + 1 \\
\text{while } \left\| \begin{bmatrix} O^{j+1} - O^j \\ \tilde{O}^{j+1} - \tilde{O}^j \end{bmatrix} \right\| > \varepsilon
\end{array}
$$

Fig. 1. Output space mapping algorithm

for the next iteration (Fig.1). The number of evaluations of the fine model is equal to the number of iterations.

Space mapping implies to choose two models, one coarse and one fine. In a system of components with heterogeneous time constants, the WRM is an adapted way to obtain a fine model with a shorter computing time than a strongly coupling model.

IV. Application

A space mapping strategy is applied to the optimization of a transformer. The device composed of a circuit supplying a transformer is considered (Fig. 2 (a)). Two models of this device are necessary to apply the space mapping. The coarse model is a circuit model of the device, where the transformer is represented as an inductance, which value is calculated using a formula. The fine model is a simulation by waveform relaxation method where the system is decomposed into two subsystems: the supply circuit (Fig. 2 (b)) and the transformer (Fig. 2 (c)). The circuit is a filter which implies that the time discretization in the circuit part has to be 400 times smaller than in the transformer part.

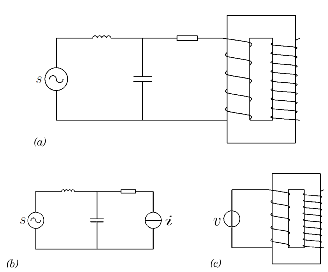

Fig. 2. (a) Complete device $(b), (c)$ WRM subsystems

The aim is to minimize the transformer mass, and to impose root mean square (RMS) current value into the transformer. The design variables are width L and height H of the transformer (Fig. 3). We denote by $i_c(t)$ and $i_f(t)$ the RMS current in the transformer, obtained respectively with the coarse and

TABLE I
Results.

	OSM	Reference
H	28.0021	28.0607
L	14.7432	14.7207
i_{rms}	0.3997	0.4008
mass	25.6706	25.6648
Nb of fine model evaluations	3	7300

the fine model. The optimization problem is:

$$
\begin{cases}
\min \{m(H, L)\}, \\
28\text{cm} \le H \le 40\text{cm}, \\
14\text{cm} \le L \le 22\text{cm}, \\
H - \frac{2L}{3} > 0, \\
i_{rms} = 0.4\ A.
\end{cases} \tag{7}
$$

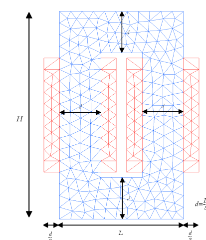

Fig. 3. Transformer geometry

OSM algorithm is applied to the optimization problem of the transformer, optimizations being executed by using genetic algorithm (GA), but the correction is applied only to the constraint on the current. By this way, the number of evaluations of a time consuming model is reduced, and a solution close to the fine problem is found (Table I).

The use of the space mapping has permitted a gain of time very substantial: 40 hours for a GA optimization with only the WRM model, 10 minutes for the optimization by space mapping. Moreover, WRM allows simulating each component with respect to its own time constant, leading to a result close to the exact solution in a short time. Time simulation is divided by 10 compared to a strong coupling.

References

[1] E. Lelarasmee, A. Ruehli, and A. Sangiovanni-Vincentelli, “The waveform relaxation method for time-domain analysis of large scale integrated circuits,” *Computer-Aided Design of Integrated Circuits and Systems, IEEE Transactions on*, vol. 1, no. 3, pp. 131 – 145, july 1982.

[2] S. Schops, H. De Gersem, and A. Bartel, “A cosimulation framework for multirate time integration of field/circuit coupled problems,” *Magnetics, IEEE Transactions on*, vol. 46, no. 8, pp. 3233 –3236, aug. 2010.

[3] Z. Guofo, W. Qiya, and R. Wanbin, “An output space-mapping algorithm to optimize the dimensional parameter of electromagnetic relay,” *Magnetics, IEEE Transactions on*, vol. 47, no. 9, pp. 2194 –2199, sept. 2011.

[4] J. Bandler, Q. Cheng, D. Gebre-Mariam, K. Madsen, F. Pedersen, and J. Sondergaard, “Em-based surrogate modeling and design exploiting implicit, frequency and output space mappings,” in *Microwave Symposium Digest, 2003 IEEE MTT-S International*, vol. 2, june 2003, pp. 1003 – 1006 vol.2.

[5] L. Encica, J. Paulides, E. Lomonova, and A. Vandenput, “Aggressive output space-mapping optimization for electromagnetic actuators,” *Magnetics, IEEE Transactions on*, vol. 44, no. 6, pp. 1106 –1109, june 2008.