A Numerically Efficient Reliability-Based Robust Optimal Design Algorithm: Application to TEAM 22

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***Abstract*— TEAM Benchmark problem 22 is a standard optimization problem in electrical engineering; therefore, the comprehensive investigation of the design characteristics under uncertainty in design variables is indispensible. In this paper, the performance robustness and constraint feasibility are integrated into a single optimization model—reliability-based robust design optimization (RBRDO). The proposed RBRDO formulation yields results that provide new alternatives to the designer.**

***Index Terms*— Reliability, reliability-based robust design optimization, robustness, TEAM 22.**

I. Introduction

One superconducting magnetic energy storage (SMES) system is shown in Fig.1. The optimal design of coils in SMES is selected as the 22nd benchmark problem for testing of electromagnetic analysis method (TEAM 22) in electrical engineering [1]. The objectives are to maintain the stored energy as close as to E_0=180 MJ with a minimal stray field. In addition, the quench condition keeping superconductivity in Fig.2 must not be violated.

Uncertainties in TEAM 22 such as manufacturing tolerances of geometric variables and perturbation compensation of a current controller cannot be ignored during design optimization. There has been a large amount of works about robust design optimization (RDO) of TEAM 22 such as the worst case optimization [2] and gradient index method [3]. Undoubtedly, they can improve the robustness of both performance and constraint functions, but the precise feasibility level of constraint is not guaranteed.

The reliability-based design optimization (RBDO) is recently introduced to the optimal design of electromagnetic device [4], [5]. The RBDO focuses on finding a reliable solution where the chance of any constraints being violated is lower than a prescribed value, whereas it does not consider the effects of uncertainty on objective functions.

It is obvious that neither RDO nor RBDO, if used individually, can ensure the design quality and the reliability simultaneously. In order to investigate comprehensive design characteristics under uncertainty, the RDO and the RBDO should be integrated [6]. However, there is very little research having been done to TEAM 22. An efficient and general approach to consider uncertainty is not available.

In this paper, the robustness analysis by the gradient index method and the reliability analysis by one sensitivity-assisted Monte Carlo simulation (SA-MCS) approach are combined into a single optimization model. For implementation of greatly improving the optimality, robustness, and reliability, a new hybrid multi-objective reliability-based robust design optimization model is proposed.

II. Reliability-Based Robust Design Optimization

A. *Optimal Characteristics under Uncertainty*

- Gradient index (GI):

In GI method, the quantitative assessment of performance robustness is calculated as the maximum absolute deviation of objective function with respect to deign variables **x**:

$$GI(\mathbf{x}) = \max\left|\partial f(\mathbf{x})/\partial x_i\right|, i = 1,\cdots,n\,. \tag{1}$$

where n is the number of design variables. It can be seen that the GI method does not need any statistical information on design variations. Therefore, it is cost-effective and easily applicable to design optimization.

- Reliability (R)

The reliability of a design describes the probability of constraints keeping in the feasible region. In the SA-MCS method [4], the reliability of a design **x** is approximated as:

$$R(g(\mathbf{x}) \le 0) = N / M \tag{2}$$

where N is the number of test designs satisfying the constraint $g(\mathbf{x})\le 0$ among M total test designs randomly generated in the uncertainty set $U(\mathbf{x})$ of design **x** [4]. Since the constraint in $U(\mathbf{x})$ is approximated using Taylor expansion assisted by the sensitivity analysis, the SA-MCS method is much more efficient than the optimization based ones. Therefore, a conventional RBDO problem is formulated as follows:

$$\begin{aligned} &\text{Minimize} \quad f(\mathbf{x}) \\ &\text{subject to} \quad R\left(g_i(\mathbf{x}) \le 0\right) \ge R_i^t,\ i = 1,\cdots,m. \end{aligned} \tag{3}$$

where R^t is a target reliability decided by a designer. The (3) concentrates on the feasibility assessment but does not attempt to minimize the variability in the objective function.

B. *Reliability-Based Robust Design Optimization*

1) Constraint RBDO assisted by GI (RBDO-GI)

To provide a better and reliable design, applying the GI to minimize the performance variation in (3), a RBDO-GI optimization model is proposed and is formulated as follows:

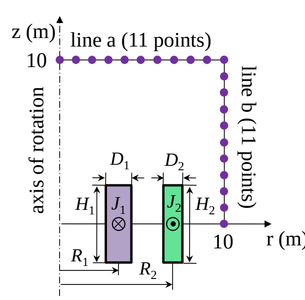


Fig. 1. Configuration of TEAM 22

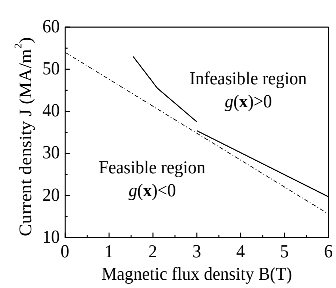


Fig.2. Quenching Condition

$$
\begin{aligned}
&\text{Minimize} \quad f(\mathbf{x}) \\
&\text{Minimize} \quad GI(\mathbf{x}) \qquad\qquad (4)\\
&\text{subject to} \quad R\left(g_i(\mathbf{x}) \le 0\right) \ge R_i^t,\ i = 1, \cdots, m.
\end{aligned}
$$

If the normal constraints replace the probabilistic ones in (4), it will become one multi-objective robust optimization problem by GI (MORO-GI). As shown in Fig.3, by enforcing probabilistic constraints at a desired level, the corresponding reliable front may be different from the Pareto front of the MORO-GI, and will be placed inside the feasible objective space. As the target reliability is increased, the front is expected to move further inside the feasible objective space.

2) Hybrid RBRDO

Considering uncertainty, it is difficult to handle multiple objectives (optimal performance, strongest robustness, and maximum reliability) without the need of weighting factors. Based on (4), applying the concept of dominance by the α-constrained method, a hybrid RBRDO model is suggested as:

$$
\begin{aligned}
&\text{Minimize} \quad GI(\mathbf{x}) \\
&\text{Maximize} \quad R_{\min}\left(g_i(\mathbf{x}) \le 0\right),\ i = 1, \cdots, m \qquad\qquad (5)\\
&\text{subject to} \quad f(\mathbf{x}) \le (1+\alpha)\% \cdot f^*(\mathbf{x})
\end{aligned}
$$

where R_{min} is the minimum reliability among all constraints, $f^*(\mathbf{x})$ is the optimal value of objective function obtained from the deterministic design optimization, and α represents the performance deviation index. Problem (5) can supply a better trade-off between robustness and reliability while the performance can be improved at the same time.

III. Optimization Result

The deterministic optimization of TEAM 22 is shown as:

$$
\begin{aligned}
&\text{Minimize} \quad f(\mathbf{x}) = \frac{B_s^2}{B_n^2} + \frac{\left|E(\mathbf{x}) - E_0\right|}{E_0} \qquad\qquad (6)\\
&\text{subject to} \quad g_i(\mathbf{x}) = \left|J_i\right| + 6.4 \cdot \left|B_{m,i}\right| - 54.0 \le 0,\ i = 1, 2
\end{aligned}
$$

where B_n=3 mT, $\mathbf{x}=[R_2, H_2, D_2]^T$, and other symbols are defined in [1]. Other models can be derived from (6).

For $f(\mathbf{x})$, uncertainty is only considered in the magnetic stray field [2]. For reliability and robust analysis, uncertain parameters of current densities independently follow Gaussian distribution as $\mathbf{J}\sim N(\boldsymbol{\mu}=\pm 22.5, \boldsymbol{\sigma}=0.23)$ MA/m^2. In the SA-MCS method, the number of test designs is 1,000,000 and the confidence level used in the uncertainty set is 0.95.

Fig.4 shows the typical Pareto fronts of MORO-GI and RBDO-GI with the target reliability of 0.9 for two constraints. It is obvious that both MORO-GI and RBDO-GI are able to search all possible designs by making a balance between $f(\mathbf{x})$ and $GI(\mathbf{x})$. Since the latter algorithm addresses reliability constraints, therefore, its Pareto front locates a little further inside feasible objective regions than that of former one as shown in Fig.4 (b). Therefore, even design A in Fig.4 (a) and design A′ are similar to the classical optimal design as listed in Table I, in fact, design A′ is more reliable and robust than design A. Fig.5 compares constraint values of $g_2(\mathbf{x})$. Undoubtedly, the Pareto solution of RBDO-GI have enough margins from constraint boundaries, which means the constraint feasibility is improved, however, some Pareto designs of MORO-GI such as design B and C almost locate on the critical boundaries with much bigger possibility to violate constraint $g_2(\mathbf{x})\le 0$.

The final optimization result of the hybrid RBRDO and discussions about will be given in the full paper.

References

[1] P. G. Alotto, *et al.*, Available: http://www.igte.tugraz.at/team 22.

[2] G. Steiner, A. Weber, and C. Magele, *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 1094-1099, Mar. 2004.

[3] N. K. Kim, *et al.*, *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 3117-3120, Aug. 2010.

[4] Z. Y. Ren, D. H. Zhang, M. T. Pham, and C. S. Koh, *Mater. Sci. Forum*, vol.721, pp.165-170, Jun. 2012.

[5] G. W. Jeung, *et al.*, *J. Magn*. vol. 17, no. 1, pp. 46-50, 2012.

[6] I. J. Lee, K. K. Choi, L. Du, and D. Gorsich, *Comput. Struct.*, pp. 1550-1562, 2008.

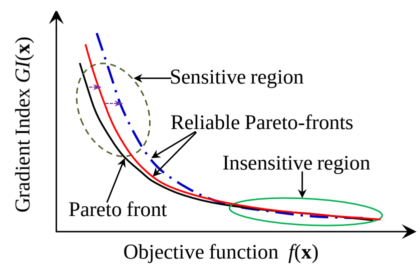
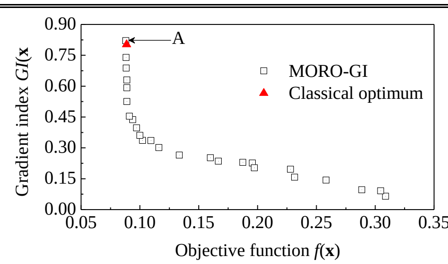


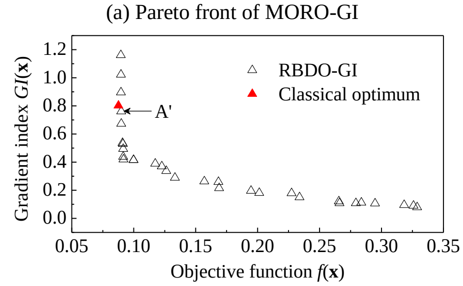
Fig. 3. Reliability-based robust design optimization assisted by GI.

TABLE I Classical Optimal design

R_2 [m]	$H_2/2$ [m]	D_2 [m]	Bs^2 [T^2]	E [MJ]	$f(\mathbf{x})$	GI
3.0819	0.2439	0.3849	7.8948E-7	180.000	8.7719E-2	0.8025



(a) Pareto front of MORO-GI



(b) Pareto front of RBDO-GI

Fig. 4. Pareto-optimal designs of *GI*-based methods.

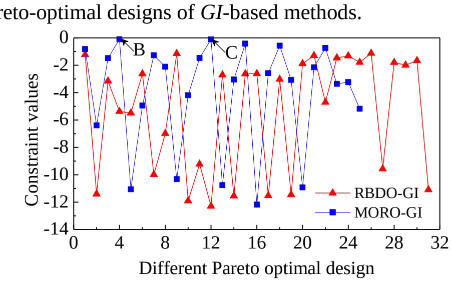


Fig. 5. Constraint comparison of $g_2(\mathbf{x})$ for different Pareto fronts where all designs have enough margins for constraint $g_1(\mathbf{x})$.