Low cogging torque design of Permanent-Magnet machine using modified multi-level set method with total variation regularization

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***Abstract*—This paper proposes a topology optimization method, which allows to design the Permanent Magnet (PM) excited machines with improved high-speed features, incorporating the topological gradient into the multi-level set method with total variation regularization. It involves the design of both the stator and the rotor, the shape of which has a predominant impact on the mechanical and electric features of a PM machine. The concept of the topological gradient is incorporated for the nucleation of new holes in the distribution of level sets. The essential advantage of the proposed method is its ability to handle topology change on a fixed mesh and its flexibility in terms of shape changing that leads to efficient computational schemes. Finally, 2/3D models are developed and deeply analyzed in order to indicate that the applied method leads to a significant reduction of both the cogging torque and the amount of higher back-EMF harmonics.**

***Index Terms*—Permanent magnet machines, electric vehicles, magnetic fields, finite element methods, rotor, stator, design optimization, level set method, harmonic analysis, topology optimization.**

I. INTRODUCTION

Permanent magnet (PM) machines have an advantage of high torque per mass and power per unit volume, as well as a relatively simple structure. Therefore, they have been widely used for industrial applications, such as robotics, computer peripherals, industrial drives or the automotive industry. In such a type of motors, the cogging torque and harmonic contents in the back-electromotive force (EMF), apart from the saturations of the magnetic circuits and converted related issue, are well-known sources of torque ripple in the developed electromagnetic torque. Hence, the minimization of cogging torque and harmonics in the back-EMF seems to be a main concern during the design process of the PM machine.

Since it is the shape of the rotor poles and the stator configuration that primarily determine the torque characteristic, this paper focuses on designing the iron and magnet rotor poles as well as the base of the tooth shape in the stator of the Electric Controlled Permanent Magnet Excited Synchronous Machine (ECPSM) [1]. Therefore, for the purposes of an optimal design problem based on the finite element analysis (FEA), a multi-level set methodology was applied in order to capture the shape variation of the material boundary on the fixed mesh, which should in turn provide a numerically efficient scheme.

II. DIRECT PROBLEM

The special construction called ECPSM is considered as a case study. Details of the construction of the machine under consideration can be found in [1].

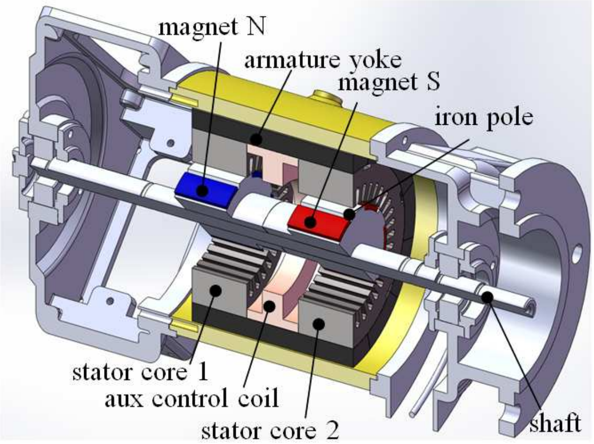


Fig. 1. Cross section of ECPSM machine with the surface-mounted PM rotor and stator structure exhibiting the three-phase single-tooth windings with the fixed excitation control auxiliary coil.

In this work, the 2D FE model, with the magnetic vector potential **A** as unknown, is used for the modelling purpose. Thus, the field distribution in the magnets and the iron parts of rotor/stator and in the air-gap can be found by solving the PDE problem defined as

$$\begin{cases} \nabla \times (\upsilon \nabla \times \mathbf{A}) & = \mathbf{0}, \\ \nabla \times (\upsilon_{PM} \nabla \times \mathbf{A}) = & \nabla \times (\upsilon_{PM}\, \mathbf{M}), \end{cases} \tag{1}$$

where υ_{PM} and $\upsilon(B)$ are the reluctivity of the permanent magnet and in the air-gap and iron, respectively, while **M** stands for the remanent flux density of the PM.

III. OPTIMIZATION PROBLEM

In practical computation, the problem of the cogging torque minimization in a 2D magnetostatic system might be reformulated into the equivalent form of the minimization of the magnetic energy variation $W_{ref}(\phi_1,\phi_2)$ as follows [2]

$$\overline{F}(\phi_1,\phi_2) = W_{ref}(\phi_1,\phi_2) + \beta_1 TV(\phi_1) + \beta_2 TV(\phi_2)$$
$$= \frac{1}{2}\int_{\Omega} \mathbf{B}(\phi_1,\phi_2)\mathbf{H}(\phi_1,\phi_2)\,d\Omega + \beta_1\int_{\Omega}|\nabla\phi_1|\,d\Omega + \beta_2\int_{\Omega}|\nabla\phi_2|\,d\Omega, \tag{2}$$

subject to the following constrain

$$B_{r-rms} = \frac{1}{L}\int_{\theta_1}^{\theta_2}[1_r B]^2 dl \le \alpha. \tag{3}$$

Here, ϕ_i signifies a signed distance functions, which represents two interfaces Γ_i between a maximum of four different regions, *TV* signifies the total variation regularization [3] with two coefficients β_1 and β_2 for controlling the complexity of the zero-level set function. B_{r-rms} is the *rms* value of magnetic field density calculated in the air-gap of the ECPS machine along the path L between the two angles θ_1 and θ_2. The total derivative of magnetic energy [4] is derived by applying the augmented Lagrangian method, the material derivative concept and the adjoint variable method

$$\frac{d\overline{F}}{d\phi_i} = \left(G_i(\mathbf{B},\mathbf{M})\Delta W' - \beta_i\nabla\cdot\frac{\nabla\phi_i}{|\nabla\phi_i|}\right)\delta(\phi_i), \tag{4}$$

where $G_i(\mathbf{B}, \mathbf{M})$ is a scalar function defined on a boundary, $\Delta W' = W' - W_0'$ is the difference between the co-energy values at defined rotor positions and the constant target average value, $\delta(\phi)$ is the Dirac delta function. The application of the Lagrange multiplier technique allows to impose the constraint (3). Finally, for minimizing an objective functional (2) by means of the modified multi-level set method, the Gauss-Newton algorithm was used. In order to indirectly control the length of the level sets, *TV* regularization was incorporated into the multi-level set method. Next, values of $\phi_i^{(k)}$ in *k*-th iteration were normalized within the range <-1,1>. Then, *k*-th distribution of ϕ_i was modified by taking into account the sign of the topology derivative calculated in the same iteration. Finally, such modified distribution of the signed function $\hat{\phi}_i^{(k)}$ was introduced into the FEM model only for the identified band of interfaces. In this case, the value of the topological gradients and the shape gradients differ by a factor 2 at any point of domain Ω [4], [6].

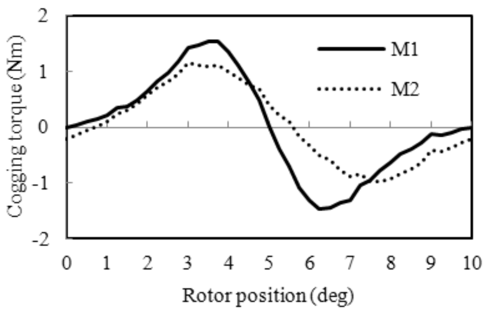
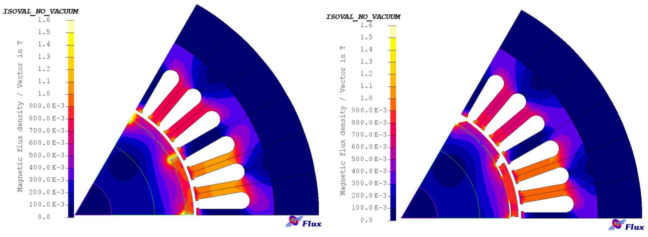


Fig. 2. Cogging torque vs. rotor position for the initial model M1 and the optimized configuration of the ECPS machine M2.

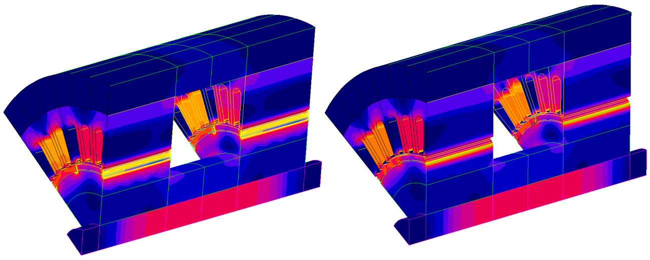
IV. RESULTS AND CONCLUSIONS

The aim of this study was to decrease the level of noise and vibration in the ECPS machine used in modern drives for electro-mobiles. Therefore, the shape of the iron pole and the part of the stator of the PM were investigated. The applied methodology, based on the multi-level set method and the topological gradient method, resulted in the reduction of the cogging torque of the initial structure of the PM machine according to the level of the magnetic flux density. This paper also underlined the unique design features of the proposed methodology. In comparison to the standard multi-level algorithm, the convergence of the presented algorithm was considerably improved. Based on the preliminary results for the simplified model, the full 3D design optimization of the new type of PM machine has to be provided.



a) b)

Fig. 3. Magnetic field distribution in 2D model. (a) Before optimization. (b) After optimization.



a) b)

Fig. 4. Magnetic field distribution in 3D model. (a) Before optimization. (b) After optimization.

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