Meaning of the Rational Solution Obtained by Game Theory

Tomoyuki Miyamoto[1], Shinya Matsutomo[2], Fumiaki Nobuyama[1], and So Noguchi[1]
1) Graduate School of Information Science and Technology, Hokkaido University
Kita 14 Nishi 9, Kita-ku, Sapporo 060-0814, JAPAN
noguchi@ssi.ist.hokudai.ac.jp
2) Department of Electronic Control Engineering, Niihama National College of Technology
7-1 Yagumocho, Niihama 792-8580, JAPAN

***Abstract*— Usually, some objectives have to be considered when an electromagnetic apparatus is optimally designed. In such a multipurposed optimal design problem many solutions exist and they are termed Pareto optimum. Hence, we have previously proposed a multipurposed optimal design method applying game theory. The proposed method can choose one rational solution from Pareto optima. It is, however, unconfirmed what the signification of the rational solution is. Therefore, the signification is investigated by calculating the weight parameters of a weighted summed objective function.**

***Index Terms*—Design optimization, electric machines, pareto optimization.**

I. Introduction

Recently an optimal design method combining numerical simulation with optimization technique is often applied on design stage of electromagnet apparatuses, such as motors, generators, and transformers [1],[2], for reducing a time, a cost, and a labor. In such an optimal design of electromagnetic apparatuses, however, multiple objectives have to be considered, and they often present conflicting and trade-off characteristics [3]. Many methods for searching for Pareto optimal solutions and exploring design space simultaneously have been proposed [4]. It is difficult to choose one valuable solution among those Pareto optimal solutions, but a single objective problem with weighted sum of the objective functions is employed instead of the multiobjective problem. However, the weight parameters cannot be decided rationally, and they are decided empirically.

To settle such a problem, we have proposed a multipurposed optimal design method applying Game Theory [5]. The proposed method was applied to the optimal design of a surface permanent magnet (SPM) motor, and it was clarified that it was possible to choose one rational solution among the Pareto optima. However, the worth and signification of the rational solution were not clarified in [5]. Therefore, in this paper, the weight parameters of weighted summed objective function are decided from the rational solution, and the worth of each single objective function is compared. The signification of the rational solution will be clarified.

II. Multipurposed Optimal Design Method

A. Optimal Design Based on Game Theory

The SPM motor is optimally designed using the previously proposed method based on the Game Theory [5]. Fig. 1 shows the schematic view of the designed SPM motor, and Tables I and II show the design variables and the SPM specifications. The objectives of the optimization problem are to reduce copper loss W_c and iron loss W_i, and these two objective functions f_1 and f_2 to be simultaneously maximized are defined as

$$f_1 = 100/W_c \text{ and } f_2 = 100/W_i. \tag{1}$$

These two objectives are conflicting and trade-off in general. A rational solution was chosen among solution candidates by the Game Theory. Fig. 2 shows the chosen rational solution with f_1 = 7.78 and f_2 = 12.68 and the solution candidates. The rational-solution selection method is detailed in [5]. The rational solution is on a Pareto front, but the signification of the rational solution was not clear in [5].

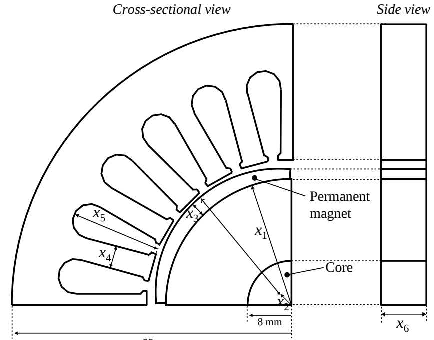

Fig. 1. Schematic view of SPM motor to be optimally designed, design variables

TABLE I
DESIGN VARIABLES

	Variables	Optimized value
Radius of rotor core	x_1 (mm)	19.0
Center position of outer arc surface of permanent magnet	x_2 (mm)	3.5
Thickness of permanent magnet	x_3 (mm)	1.95
Teeth width	x_4 (mm)	3.0
Teeth height	x_5 (mm)	20.0
Motor thickness	x_6 (mm)	65.0
Wire diameter	x_7 (mm)	1.25
Phase of supply voltage	x_8 (deg.)	13.0

TABLE II
DESIGN SPECIFICATIONS

Rotated torque	1.7 Nm
Rotated speed	4000 rpm
Residual magnetic flux density of permanent magnet	1.2 T
Space factor of wire in slot	under 60%

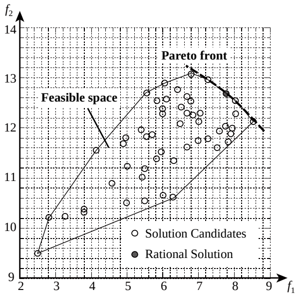

Fig. 2. Rational solution and solution candidates in Game theory.

B. *Signification of the Rational Solution*

For multipurposed optimization, a weighted summed objective function is usually employed. It is, however, difficult to decide the weight parameters even if the objectives are not conflicting and trade-off. Therefore, the optimal design method applying the Game Theory was applied to the SPM motor design problem. Here, in order to investigate the signification of the rational solution obtained, the weight parameters of a weighted summed objective function are decided below.

The weighted summed objective function F to be minimized is defined as

$$F = w_1 \frac{1}{f_1} + w_2 \frac{1}{f_2} \qquad (w_1 + w_2 = 1) \tag{2}$$

where w_1 and w_2 are the weight parameters of the weighted summed objective function. Here, it is supposed that a Pareto front is linear and represented by

$$f_2 = af_1 + b \tag{3}$$

where a and b are the parameters. Since the rational solution is on the Pareto front, (3) is substituted into (2). The differential of F becomes 0 for minimizing F, as follows:

$$F'(f_1) = \frac{a(w_1 - 1)}{(b + af_1)^2} - \frac{w_1}{f_1^2} = 0. \tag{4}$$

The weight parameter w_1 is obtained from (4), as follows:

$$w_1 = \frac{af_1^2}{af_1^2 - (b + af_1)^2}. \tag{5}$$

In the SPM motor optimal design problem, a and b are -0.55 and 16.95, respectively. Since the solution of (2) corresponds to the rational solution obtained above, the weight parameters are obtained, w_1 = 0.171 and w_2 = 0.829. From the obtained weight parameters, it is possible to grasp the signification of the rational solution. In the optimal design problem of the SPM motor, the objective function f_1 is much less worthy than f_2.

III. Optimal Design with the Obtained Weight Parameters

The weight parameters of the rational solution obtained with the Game Theory is calculated. Hence, the optimal design of the SPM motor is re-performed by solving (2) with the newly obtained weight parameter. Fig. 3 shows the result of the optimal shape of the SPM motor, and Table III shows the design variables of the newly optimized SPM motor.

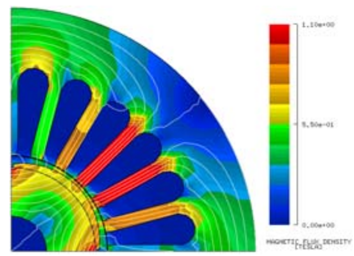
Fig. 3. Newly optimized configuration of SPM motor using the weighted summed objective function.

TABLE III
DESIGN VARIABLES

	Variables	Optimized value
Radius of rotor core	x_1 (mm)	19.60
Center position of outer arc surface of permanent magnet	x_2 (mm)	3.72
Thickness of permanent magnet	x_3 (mm)	1.61
Teeth width	x_4 (mm)	2.76
Teeth height	x_5 (mm)	17.50
Motor thickness	x_6 (mm)	58.90
Wire diameter	x_7 (mm)	2.16
Phase of supply voltage	x_8 (deg.)	17.10

IV. Conclusion

We have proposed a way to decide weight parameters of a weighted summed objective function so that an optimal solution corresponds to a rational solution obtained from the Game Theory. From the obtained weight parameters, it is possible to infer the signification of the rational solution. Although the Game theory can deal with discrete variables in a very narrow exploring space, a common optimization method can deal with continuous variables in a wider space. Therefore, using the obtained weight parameters, it is possible to re-explore an optimal solution in the wide space by a common optimization method as a single objective problem.

References

[1] K. Watanabe, F. Campelo, Y. Iijima, K. Kawano, T. Matsuo, T. Mifune, and H. Igarashi "Optimization of inductors using evolutionary algorithms and its experimental validation," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 3393-3396, Aug. 2010.

[2] N. Takahashi, T. Yamada, S. Shimose, and D. Miyagi, "Optimization of Rotor of Actual IPM Motor Using ON/OFF Method," *IEEE Trans. Magn.*, vol. 47, no. 5, pp. 1262-1265, May 2011.

[3] L. S. Coelho, L. Z. Barbosa, and L. Lebensztajn, "Multiobjective Particle swarm Approach for the Design of a Brushless DC Wheel Motor," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 2994-2997, Aug. 2010.

[4] C. A. C. Coelle, G. T. Pulido, and M. S. Lechuga, "Handling multiple objectives with particle swarm optimization," *IEEE Trans. Evo. Comp.*, vol. 8, no. 3, pp. 256-279, 2004.

[5] T. Miyamoto, S. Noguchi, and H. Yamashita, "Selection of an optimal solution for multiobjective electromagnetic apparatus design based on game theory," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 1026-1029, Jun. 2008.