Statistical Moment-based Robust Design Optimization for Nonlinear Electromagnetic Devices

Su-gil Cho, Junyong Jang and Tae Hee Lee, Member, IEEE
Department of Automotive Engineering, Hanyang University, 222 Wangsimni-ro, Seoul 133-791, Korea
thlee@hanyang.ac.kr

***Abstract*—Robust design has gained much attention in product design because it can find the best design solution by minimizing the variance of response due to the variances of the variables which are impossible to control. However, since previous robust design techniques were inaccurate, they are hard to apply to product design with nonlinear properties such as electromagnetic devices. Thus, in this paper, statistical moment-based robust design optimization is proposed to resolve the difficulty. The proposed method can search for a robust optimum design more accurately as well as more efficiently since this calculates mean and deviation of the system directly from multiplicative decomposition method without any assumptions. Electromagnetic device is used to demonstrate its feasibility.**

***Index Terms*—Design optimization, Magnetic devices, Moment methods, Robustness, Uncertainty.**

I. Introduction

Deterministic optimization has been widely applied to engineering design. However, deterministic approaches do not consider the impact of unavoidable uncertainties associated with design parameters and design variables in any engineering system. Hence deterministic optimal solutions may be sensitive with the presence of uncertainty. Robust design optimization is one of the advanced techniques that can improve product quality to find the non-sensitive solution, i.e. robust solution, without eliminating the causes. Despite this benefit of robust design, it faces challenging issues of inefficiency and inaccuracy.

Many methods have been developed for modeling of robust design problems [1]. Taguchi method, proposed by Taguchi, has been widely used in robust design because of easy implementation. However, the method cannot provide design solutions but only the direction to the optimum solution. Also it is inadequate to implement nonlinear system such as electromagnetics devices due to inaccuracy. To solve this problem, the first-order Taylor expansion has been developed though efficient but inaccurate. Monte Carlo simulation is another choice, it is accurate but inefficient.

In this paper, statistical moment-based robust design optimization is proposed to calculate the mean and variance (or standard deviation) of the objective and constraint functions (response variables). The proposed method can search for a robust optimum design more accurately as well as more efficiently since this calculates mean and deviation of the system directly from multiplicative decomposition method (MDM) based on metamodel without any assumptions [2]. Electromagnetic example (TEAM Workshop Problem 22) is used to demonstrate the effectiveness of the proposed method. The results are compared with those from deterministic optimization, Taylor expansion method and Monte Carlo simulation in terms of accuracy and efficiency.

II. Statistical Moment-based Robust Design

As shown in Fig. 1, there is a distinct difference between the deterministic design and robust design. In deterministic design, point 1 is selected as the optimum solution, i.e the minimum value in design space. However, if variables have tolerance which is called variation of variables, the optimum point has potential to violate the constraint. Robust design, on the other hand, chooses the point 2 as the optimum solution that can obtain robust performance without change in tolerance range. Also it never violates the constraint.

When variables have tolerance, the objective and the constraint functions inevitably have distribution due to tolerance. Generally in robust optimization, the objective and constraint functions are redefined as mean and deviation of original objective and constraint functions. Design formulation for robustness is represented as follows:

$$\text{Find } \mathbf{b} \in R^n$$

$$\text{To minimize } F(\mathbf{b}) = w_1 \frac{\mu_f(\mathbf{b}+\mathbf{P})}{\mu_f^*} + w_2 \frac{\sigma_f(\mathbf{b}+\mathbf{P})}{\sigma_f^*} \tag{1}$$

$$\text{Subject to } G_i(\mathbf{b}) = \mu_{g_i}(\mathbf{b}+\mathbf{P}) - k\sigma_{g_i}(\mathbf{b}+\mathbf{P}) \le 0$$

$$\mathbf{b}_L \le \mathbf{b} \le \mathbf{b}_U,\ i = 1,2,\cdots,l$$

Where $\mathbf{P}$ is the noise variables which are uncontrollable and distributed. The design variables represented by $\mathbf{b}$ are selected as design variables that can be controlled by the designer and have significant impacts on the problem characteristics. F and G_j are the functions redefined as mean and deviation of original objective and constraint functions respectively. w_i is weighting factor between mean and variance and μ_f^*, σ_f^* are normalized factors of mean and variance.

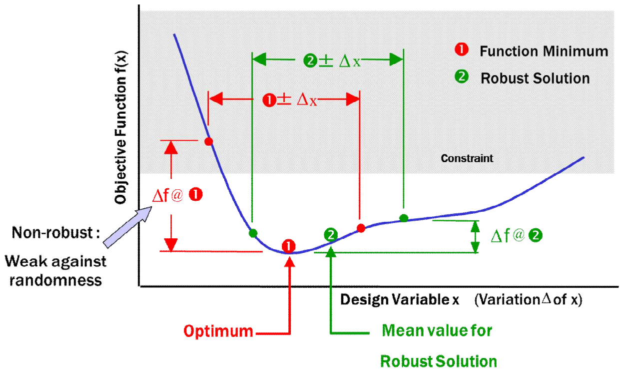

Fig. 1. Concept of robust design and general optimum design

III. Multiplicative Decomposition Method

The statistical moments in Eq. 2 can be expressed in terms of the first two *raw moments* as follows:

$$m_l = \int_{\Omega} Y(\mathbf{x})^l f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}, \quad (l = 1,2) \tag{2}$$

Fundamental idea behind MDM is to replace true response $Y(\mathbf{x})$ with kriging metamodel, $\hat{Y}(\mathbf{x})$. Kriging, so-called design and analysis of computer experiment (DACE) model, is the interpolation model where the prediction coincides with the simulation response at sampled points exactly. If kriging metamodel is accurate enough to replace true function, the first raw moment can be expressed as

$$\begin{aligned} m_1 &\cong \int_{\Omega} \hat{Y}(\mathbf{x}) f_X(\mathbf{x}) d\mathbf{x} \\ &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \{\hat{\beta} + \mathbf{r}(\mathbf{x})^T \mathbf{R}^{-1}(\mathbf{Y} - \mathbf{F}\hat{\beta})\} \prod_{k=1}^{D} \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{1}{2}\left(\frac{x_k-\mu_k}{\sigma_k}\right)^2} dx_1 \cdots dx_D \\ &= \hat{\beta} + \mathbf{J}^T \mathbf{R}^{-1}(\mathbf{Y} - \mathbf{F}\hat{\beta}) \end{aligned} \tag{3}$$

$$\mathbf{J} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \mathbf{r}(\mathbf{x}) \prod_{k=1}^{D} \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{1}{2}\left(\frac{x_k-\mu_k}{\sigma_k}\right)^2} dx_1 \cdots dx_D$$

where μ_k and σ_k are mean and standard deviation of k-th design variable. In Eq. 3, only correlation vector $\mathbf{r}(\mathbf{x})$ needs to be explicitly integrated because the others become constant. It is important to note that integration of each PDF over the infinite region becomes one. Therefore, integral of i-th element of $\mathbf{J}$ vector can be given as

$$\begin{aligned} J_i &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{k=1}^{D} \left(e^{-\theta_k (x_k - s_k^i)^2} \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{1}{2}\left(\frac{x_k-\mu_k}{\sigma_k}\right)^2} \right) dx_1 \cdots dx_D \\ &= \left(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\theta_1 (x_1 - s_1^i)^2 - \theta_1' (x_1 - \mu_1)^2} dx_1 \right) \cdots \left(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_D} e^{-\theta_D (x_D - s_D^i)^2 - \theta_D' (x_D - \mu_D)^2} dx_D \right) \end{aligned} \tag{4}$$

where new variables $\theta_k' = 1/(2\sigma_k^2)(k = 1,2,\cdots,D)$ are used for simplicity of equation.

In Eq. 4, we can find out an important fact that D-dimensional integral is decomposed into product of one-dimensional integrals. Because the integral in each dimension can be explicitly calculated, it does not need to use a numerical integration scheme [2]. Thus, we can accurately obtain mean and variance of responses by using MDM.

IV. TEAM Workshop Problem 22

TEAM workshop problem 22 is classified as three- and eight-parameter problems [3]. In this paper, we consider the three design parameter problem. The objective function of the problem takes into account both the energy requirement (E should be as close as possible to 180 MJ) and the minimum stray field requirement. And the superconducting material should not violate the quench condition that links together the value of the current density and the maximum value of magnetic flux density. In this problem, because SMES device is a highly elaborate machine, it is required a high level of accuracy during manufacturing process. However, uncertainty of currents in two coils which is hard to control has influence on energy preservation [4]. Therefore, it is necessary to employ the robust design optimization for stable performance. Design variables, limits of those and uncertainties of currents in two coils are specified in Table I.

TABLE I
DESIGN VARIABLES AND ITS UNCERTAINTY

Limits	R_1 [m]	R_2 [m]	$h_1/2$ [m]	$h_2/2$ [m]	d_1 [m]	d_2 [m]	J_1 $\left[\frac{\text{MA}}{\text{m}^2}\right]$	J_2 $\left[\frac{\text{MA}}{\text{m}^2}\right]$
Lower	1.0	1.8	0.1	0.1	0.1	0.1	10	-10
Upper	4.0	5.0	1.8	1.8	0.8	0.8	30	-30
Deviation	-	-	-	-	-	-	0.1	0.1

TABLE II
MAIN SPECIFICATIONS AND REQUIREMENTS OF IPMSM

	Deterministic optimum	Robust optimum
Design variable		
Mean of objective f		
Deviation of objective f		
Total F		

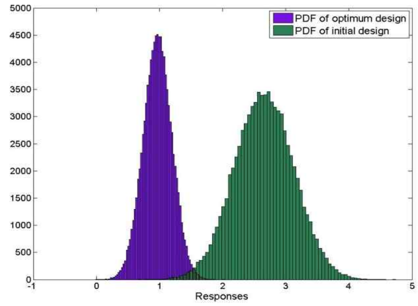

Fig. 2. Comparison of the PDF at initial design and optimum design using Monte Carlo simulation

V. Result

Electromagnetic analysis of TEAM problem 22 is performed by ANSYS 12.0, FE program. Latin hypercube sampling technique (50 pre-sampled points) is used as each iteration sample set. Sequential optimization is performed gradient-based algorithm in MATLAB. Finally, we can find robust optimum at 14th iteration which satisfies constraints. Comparison between robust and deterministic optimum is presented as a histogram in Fig. 2.

VI. Conclusion

In this paper, we formulate robust problem of SMES system and it is able to evaluate robustness of the system due to uncertainty of currents in two coils. From optimization result, we obtain the robust optimum solution that improves both the performance and robustness of SMES devices.

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