Design Improvement for Cogging Torque Reduction in Axial-Flux Permanent-Magnet Machines Using Schwarz-Christoffel Transformation

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***Abstract*— This paper presents a fast and accurate method for calculating magnetic field in axial flux permanent magnet machine with slotted structure. Schwarz-Christoffel transformation is used for obtaining normal (axial) and tangential components of air-gap flux density and then cogging torque calculated by integrating the Maxwell stress tensor inside the air-gap. We used complex relative air-gap permeance obtained using SC Toolbox to reduce computation time that is useful for design optimization process. The accuracy of the developed method is verified using 3-D finite element method.**

***Index Terms*—Axial flux machine, Schwarz-Christoffel transformation, cogging torque.**

I. Introduction

The accurate performance of Axial-Flux Permanent-Magnet (AFPM) machines can be evaluated by three-dimensional (3-D) finite element method (FEM), however 3-D FEM analysis is very time consuming. Cogging Torque reduction of AFPM machines has been one of interest in many researchers and a variety of techniques such as displacing and shaping the magnets, skewing the slots and/or magnets are used [1]. To take into account 3-D intrinsic nature of AFPM machine, quasi-3-D method is used [2].

Schwarz-Christoffel (SC) transformation is an attractive and powerful conformal mapping (CM) that is based on the complex mathematical theory. Recently MATLAB SC Toolbox is developed [3] and has been used in several researches for numerical mapping. [4], [5]. AFPM machine (Fig. 1), with parameters given in Table I is used as the test machine.

II. Air-Gap Field Calculation

The accurate knowledge of magnetic field in air-gap is essential for accurate prediction of the motor performance. It can be shown that the magnetic field produced both inside and outside of a PM having magnetization M, is exactly the same as what would be produced by two equivalent currents, volume current with the density $\vec{J}_m = \vec{\nabla} \times \vec{M}$ and surface current with the density $\vec{J}_{ms} = \vec{M} \times \vec{n}$, where $\vec{M}$ is magnetization vector and $\vec{n}$ is a unit vector normal to surface of the PM [6]. As PM having uniform magnetization in z direction ($\vec{M} = M\vec{z}$), then $\vec{\nabla} \times \vec{M} = 0$. Therefore, there is no volume equivalent current. The equivalent surface current exist along lateral edge of PM. Surface currents divide into a discrete number of punctual current as shown in Fig. 4. The current that flows from each of these, is;

$$I = \frac{h_m \, M}{n'} \tag{1}$$

where n' is the number of punctual currents and h_m is the thickness of PM. The magnetization M in operating point can be found by solving simple equivalent magnetic circuit as;

$$M = \frac{Br}{\mu_0} \frac{h_m + g}{h_m + \mu_r \, g} \tag{2}$$

For field calculation, first magnetic field is calculated in slot-less air-gap in T-domain Fig. 3. In configuration where punctual currents are placed in the air-gap between two infinite iron planes, the flux density is given by [5];

$$B_t = \sum_{m=1}^{n'} \frac{j\mu_0 I_m}{4(g+h_m)} \left[\coth\left(\frac{\pi (t - t_{\mathrm{Im}})^*}{2(g+h_m)} \right) + \coth\left(\frac{\pi \left(t - t_{\mathrm{Im}}{}^*\right)^*}{2(g+h_m)} \right) \right] \tag{3}$$

where g is air-gap length, t is coordinate of the point that field is to be calculated, and t_{Im} is the m_{th} punctual current coordinate. Flux density in W-domain can be obtained as [7];

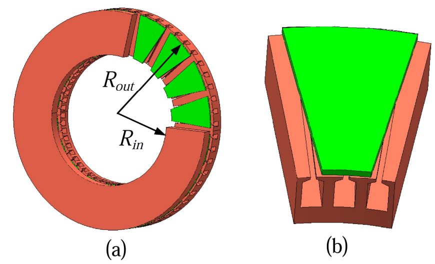


Fig. 1. (a) AFPM machine. (b) PM shape.

TABLE I
Parameters Of the Test AFPM Machine

Symbol	*parameter*	*Value*	*Unit*
P	Number of pole-pairs	9	-
Q_s	Number of stator slots	54	-
R_{out}	External radius of machine	241	mm
R_{in}	Internal radius of machine	144.5	mm
g	Physical length of air-gap	1.5	mm
h_m	Axial thickness of PM	4	mm
B_r	PMs remanent	1.144	T
μ_r	Relative recoil permeability	1.1	-
b_{s0}	Slots opening width	3	mm

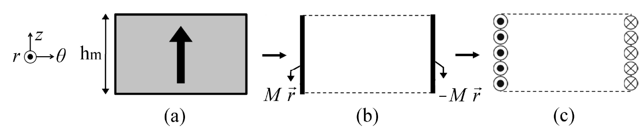
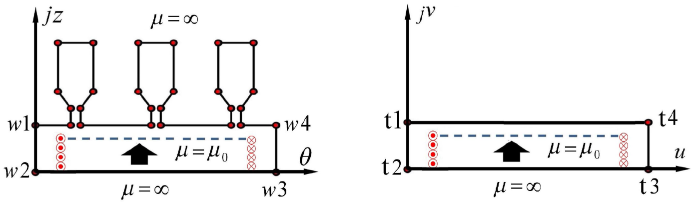


Fig. 2. PM equivalent currents, (a) actual PM. (b) PM equivalent surface currents. (c) PM equivalent punctual currents



(a) (b)

Fig. 3. Representation of one pole pitch of air-gap and equivalent currents in (a) W-domain and in T-domain (b).

$$B_w = B_\theta + jB_z = \left(\frac{\partial t}{\partial w}\right)^* B_t \tag{4}$$

where $\partial t / \partial w$ is complex relative air-gap permeance and obtained using SC Toolbox. The real and imaginary parts of relative air-gap permeance and flux density in both domains for one of the layers in quasi3-D method are shown in Fig. 4.

III. Cogging Torque Minimization

The cogging torque for each layer in quasi-3-D method is calculated by integrating the Maxwell stress tensor inside the air-gap.

$$T_{C,i} = \frac{2P\,R_{ave,i}^2\,(R_{out} - R_{in})}{\mu_0\,N_l}\int_0^{\frac{\pi}{P}} B_{z,i}\,B_{\theta,i}\,d\theta \tag{5}$$

where N_l is the number of layers used in quasi-3-D computation, and R_{ave} is average radius of i_{th} layer. The total cogging torque of machine is obtained as;

$$T_C = \sum_{i=1}^{N_l} T_{C,i} \tag{6}$$

The tooth width is small at R_{in} and become large at R_{out}. So, to avoid saturation of tooth tip and extra core loss, we use PM shape which PM-width to pole-width ratio α_{PM} gradually increases along the machine radius, as shown in Fig. 1. So α_{PM} in R radius of machine is defined as;

$$\alpha_{PM}(R) = \frac{R}{R_{out}} k_m \tag{7}$$

where k_m is α_{PM} value at outer radius. Using proposed method, cogging torque is obtained for k_m value from 0.75 to 0.95 and the results are compared with results obtained from 3-D FEM as shown in Fig. 5. It can be seen that peak of cogging torque reaches a minimum when $k_m = .85$.

IV. Conclusion

In this paper, SC transformation technique has been used for calculating cogging torque in AFPM machine with slotted

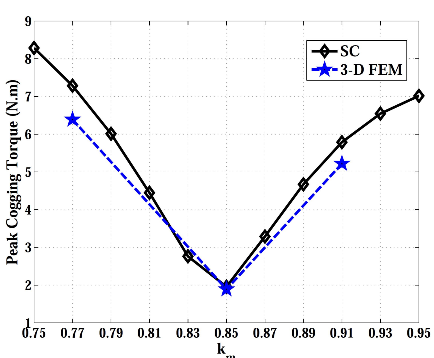


Fig. 5. Influence of k_m on peak cogging torque.

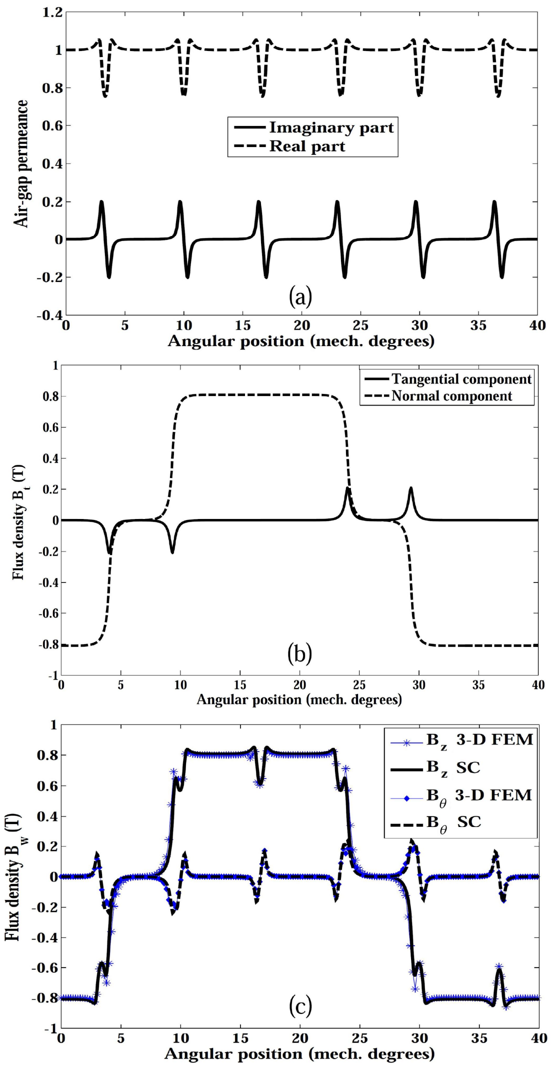


Fig. 4. (a) Air-gap relative permeance. (b) T-domain flux density. (c) W-domain flux density.

structure. This method is ideal for use in the design process aiming at cogging torque minimization.

References

[1] M. Aydin, Z. Q. Zhu, T. A. Lipo, and D. Howe, "Minimization of cogging torque in axial-flux permanent-magnet machines: design concepts," *IEEE Trans. Magn.*, vol. 43, no. 9, pp. 3614–3622, Sep. 2007.

[2] H. Tiegna, A. Bellara, Y. Amara, and , G. Barakat, "Analytical modeling of the open-circuit magnetic field in axial flux permanent magnet machines with semi-closed slots," *IEEE Trans. Magn.*, vol. 48, no. 3, pp. 1212–1226, Mar. 2010.

[3] T. A. Driscoll, "Algorithm 843: Improvements to the Schwarz–Christoffel Toolbox for MATLAB," *ACM Trans. Math. Softw.*, vol. 31, pp. 239–251, 2005.

[4] T. C. O'Connell and P. T. Krein, "A Schwarz–Christoffel-based analytical method for electric machine field analysis," *IEEE Trans. Energy Convers.*, vol. 24, no. 3, pp. 565–577, Sep. 2009.

[5] K. Boughrara, D. Zarko, R. Ibtiouen, O. Touhami, and A. Rezzoug, "Magnetic field analysis of inset and surface-mounted permanent-magnet synchronous motors using Schwarz-Christoffel transformation," IEEE Trans. Magn., vol. 45, no. 8, pp. 3166–3178, Aug. 2009.

[6] M. Markovic, M. Jufer, and Y. Perriard, "Analyzing an electromechanical actuator by Schwarz-Christoffel mapping," *IEEE Trans. Magn.*, vol. 40, no. 4, pp. 1858–1863, Jul. 2004.

[7] D. Zarko, D. Ban, and T. A. Lipo, "Analytical calculation of magnetic field distribution in the slotted air gap of a surface permanent-magnet

motor using complex relative air-gap permeance," *IEEE Trans. Magn.*, vol. 42, no. 7, pp. 1828–1837, Jul. 2006.