Representation of Electrical Machine Windings using $\boldsymbol{T}_0$ Formulation

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***Abstract*— In the paper the representation of electrical machine stranded windings using T_0 formulation has been presented. In the discussed method the electric vector potential T_0 has been applied. In the proposed approach the distribution of the potential T_0 inside the winding is determined for given distribution of T_0 on the boundaries of the winding subdomains. Usefulness of proposed method has been tested on the example of multi-turn coil of two layer winding of the high voltage asynchronous machine stator.**

***Index Terms*—Numerical analysis, electromagnetic devices, electric machines, coils, finite element method.**

I. Introduction

The one of the problems presently being under examination in the area of FEM modeling of electrical machines is proper and accurate representation of the windings that are made of stranded conductors [1], [2]. In such windings the conductors are treated as thin wires where skin effects are neglected. The most commonly used methods of the representation of this kind of windings in the space of finite element analysis base on implementation of the Biot-Savart law [2],[4]. The advantage of those methods is simple algorithm of determining field sources. However implementation of those algorithms characterize rather high computational costs especially in the windings with geometrically complicated shape. An interesting way of stranded coils representation in finite element space has been presented and discussed in [1]. In this contribution author proposes two methods: (a) method based on definition of number of wires (turns) passing through finite element faces; and (b) method, in which winding distribution is defined by crosses of element edges and surfaces formed by turns of the coil. As author demonstrates the proposed methods are universal [1], however the complexity of automation process of determining field sources highly depend on number of turns in the winding. Other interesting methods have been proposed in [3],[5]. To represent windings and determine field sources authors of those papers propose to define and use the turn density vectors. Typically these vectors are calculated on basis of supposed current density distribution $\boldsymbol{J}_0$. Nevertheless the examples presented in the contributions relates only to coils with simple geometry, where current density distribution $\boldsymbol{J}_0$ can be described by analytical functions.

In this paper the method of electrical machine stranded winding description using electric vector potential $\boldsymbol{T}_0$ [6] has been presented and discussed. The method has been explained on the basis of example of a multi-turn coil shown in the Figure 1. In the proposed approach the distribution of the potential $\boldsymbol{T}_0$ inside the winding is determined for given distribution of $\boldsymbol{T}_0$ on the boundaries of the winding sub-domains. The major advantage of discussed method is versatility and usefulness in the description simple and also geometrically complex coils. Moreover by using proposed method the field sources for formulations using either scalar Φ or vector $\boldsymbol{A}$ potentials can be easily determined. The correctness and accuracy of the method has been tested on the example of the two layers winding coil of asynchronous motor working in cryogenic conditions [7].

II. Description of the Winding Using $\boldsymbol{T}_0$ Formulation

In the considered domain Ω two sub-domains have been isolated: (a) sub-domain Ω_c that simple shaped represent multi-turn coil with turn number equals z_c; and (b) sub-domain Ω_n, without currents. Considered domain Ω has been shown in the Figure 1. Between sub-domains (a) and (b) two boundary surfaces Γ_o i $\Gamma_{\boldsymbol{T}o}$ have been defined. That surfaces disintegrate considered sub-domains and boundary surface Γ_Ω of domain Ω. Therefore the coil sub-domain is treated as a multiple connected region.

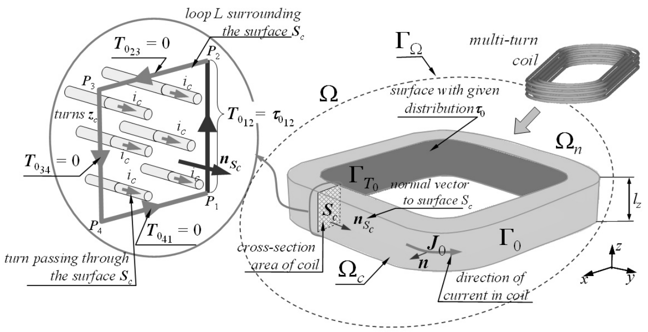

Fig.1. A considered multi-turn coil

For given above conditions the distribution of electric vector potential $\boldsymbol{T}_0$ for considered domain can be found by solving following equations

$$\nabla \times \boldsymbol{\rho} \nabla \times \boldsymbol{T}_0 = 0 \big|_{\Omega_c} \tag{1a}$$

in sub-domain Ω_c, and

$$\nabla \times \nabla \times \boldsymbol{T}_0 = 0 \big|_{\Omega_n} \tag{1b}$$

in sub-domain Ω_n, for defined boundary conditions

$$\boldsymbol{T}_0 = \boldsymbol{\tau}_0 \big|_{\Gamma_{To}} \tag{2a}$$

$$\boldsymbol{T}_0 = 0 \big|_{\Gamma_0 + \Gamma_\Omega} \tag{2b}$$

where: $\boldsymbol{\rho}$ is tensor of resistivity and $\boldsymbol{\tau}_0$ is function that describe distribution of potential $\boldsymbol{T}_0$ on the surface $\Gamma_{\boldsymbol{T}o}$.

Equations (1) have been derived on basis of the DC current conduction field for domain Ω and relation between current density vector $\boldsymbol{J}_0$ and electric vector potential $\boldsymbol{T}_0$, i.e. $\boldsymbol{J}_0 = \nabla\times\boldsymbol{T}_0$. Taking into account the boundary conditions (2) in (1) the distribution of potential $\boldsymbol{T}_0$ inside domain Ω can be determined from

$$\nabla\times\boldsymbol{\rho}\nabla\times\boldsymbol{T}_0\,\big|_{\Omega_c} + \nabla\times\boldsymbol{\rho}\nabla\times\boldsymbol{\tau}_0\,\big|_{\Gamma_{T_0}\rightarrow\Omega_c} = 0 \qquad (3a)$$

for sub-domain Ω_c, and

$$\nabla\times\nabla\times\boldsymbol{T}_0\,\big|_{\Omega_n} + \nabla\times\nabla\times\boldsymbol{\tau}_0\,\big|_{\Gamma_{T_0}\rightarrow\Omega_n} = 0 \qquad (3b)$$

for sub-domain Ω_n.

The boundary function $\boldsymbol{\tau}_0$ can be easily determined by numerical solving a condition that normal component of current density $\boldsymbol{J}_0$ at considered boundary surface $\Gamma_{\boldsymbol{T}0}$ of coil sub-domain Ω_c is equal zero, i.e.

$$(\nabla\times\boldsymbol{\tau}_0)\circ\boldsymbol{n} = 0 \qquad (4)$$

where $\boldsymbol{n}$ is the normal vector to the considered boundary surface. To solve (4) it is necessary to satisfy the relation between magnetomotive force θ, vector $\boldsymbol{J}_0$ passing through cross-section of the coil $\boldsymbol{S}_c$ perpendicular to current flow direction and vector $\boldsymbol{T}_0$ along loop L surrounding surface $\boldsymbol{S}_c$ (Fig. 1). That relation can be expressed in the following form

$$\theta = z_c i_c = \int_{S_c} \boldsymbol{J}_0\,\mathrm{d}\boldsymbol{s} = \oint_{L(S_c)} \boldsymbol{T}_0 \mathrm{d}\boldsymbol{l} = \boldsymbol{\tau}_{0_{12}} l_z \qquad (5)$$

where i_c is the coil current, $\boldsymbol{\tau}_{012}$ is the value of boundary function assigned to the edge P_1P_2 of loop L (Fig.1), l_z is the length of the edge P_1P_2 corresponding to the coil height.

III. Example

As mentioned the correctness and accuracy of the method have been tested on the example of the two layers winding coil (Fig. 2) of asynchronous motor working in cryogenic conditions [7]. In the considered example the current density distribution $\boldsymbol{J}_0$ inside the coil was unknown. The distribution of $\boldsymbol{J}_0$ has been determined on the basis of distribution $\boldsymbol{T}_0$ calculated by solving (3) for given boundary conditions (2). The distribution of boundary function $\boldsymbol{\tau}_0$ describing potential distribution for chosen surface has been calculated numerically on the basis of (4) and (5). The calculations have been performed for rated coil current i_c equals 32 Ampere and turn number z_c equals 20.

In order to solve (3) the edge element method (EEM) has been applied [8]. Considered region has been meshed using tetrahedron elements. The number of tetrahedron elements in the presented example was about 24 000 elements and the number of EEM equations being solved was equal 33 554. Obtained distribution of current density has been shown in the Figure 2. In order to verify correctness and accuracy of obtained results of distributions $\boldsymbol{J}_0$ two numerical tests have been proposed and performed: (a) first based on checking current flow continuity $\nabla\circ\boldsymbol{J}_0 = 0$, and second test (b) in which the magnetomotive forces calculated for different cross-sections perpendicular to current flow of the coil have been compared to each other. For test (a), the result of the numerical integration of the expression $\nabla\circ\boldsymbol{J}_0$ over whole considered domain was on the level of 10^{-13}, whereas for test (b) the identical results for chosen cross-sections of the coil have been obtained the with 7-decimal–place accuracy. On the basis of obtained results it can be said that proposed method gives sufficient accuracy.

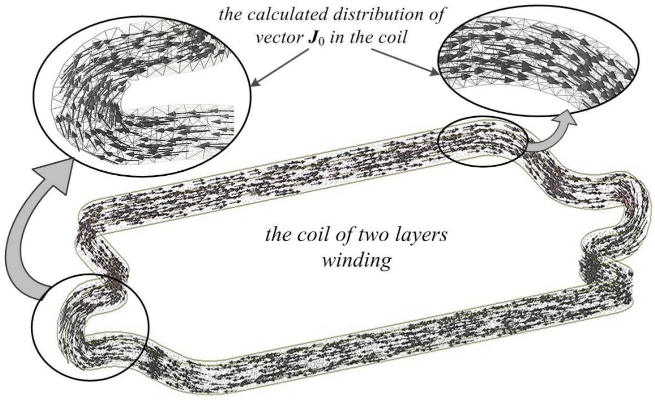

Fig.2. Distribution of current density vector $\boldsymbol{J}_0$ in coil of the double layer winding

IV. Conclusion

The method of the description of electrical machines stranded windings has been discussed. The approach base on determining the electric vector potential $\boldsymbol{T}_0$ distribution inside the coil for given its distribution on the boundaries of sub-domains. The way of defining boundary function for known value of magnetomotive force in the coil has been proposed. The major advantage of presented method of winding description in finite element space is the versatility. Proposed $\boldsymbol{T}_0$ approach can be successfully applied either for simple as well as for geometrically complex coils and it can be successfully applied for field sources description in vector and scalar potential formulations. The usefulness of considered method for complex shape coil has been proved on the basis of presented example. Obtained results, performed tests and analysis prove accuracy of the proposed method.

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