Three-dimensional Computation of Magnetic Fields in hysteretic media with time-periodic sources

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***Abstract*—In this paper, we present a 3D numerical model of an integral formulation for the efficient computation of 3D magnetic fields in hysteretic media with (time) periodic sources. The hysteretic medium is described by an isotropic vector generalization of the classical scalar Jiles-Atherton model. The nonlinear algebraic system is solved by means of Picard-Banach iterations in the frequency domain. The full matrices arising after the discretization of the main (linear) operators are suitably sparsified and distributed among the nodes of a parallel computer system. The numerical model is applied for the Non Destructive micro-magnetic characterization (of mechanical properties) in metallic materials. In particular the problem of estimating incremental permeability is analyzed.**

***Index Terms*— magnetic field problems, nondestructive evaluation, integral methods, vector hysteresis, finite element analysis, harmonic analysis.**

I. Introduction

Material characterization is a field of paramount importance in the steel industry. Obtaining information on the mechanical properties of a specimen without resorting to expensive destructive tests is a major challenge for Electromagnetic Non Destructive Evaluation (E'NDE) methods, which rely on the concept that the material microstructure affects both the mechanical and the magnetic properties of ferromagnetic materials. Specifically, the micro-magnetic characterization of mechanical properties is achieved by means of the measurements of the incremental permeability along a hysteresis loop, multi-frequency eddy currents induced in the hysteretic medium, Barkhausen noise and harmonic analysis [1]. Several experimental studies and their application in industrial environments have already demonstrated the validity of this approach [1], [2]. The design and optimization of the probe, as well as a reliable interpretation of the measurements, require accurate numerical models. Among these, a key role is played by the magneto-quasi-stationary Maxwell equations in hysteretic media with time-periodic sources.

The computation of magnetic fields in hysteretic media is a challenging problem that is of interest for numerous applications , such as the analysis of electrical machines [3], magnetic recording [4] and Non Destructive Evaluation [5]. In the latter paper we presented an integral formulation of the problem with the hysteretic medium described by an isotropic vector generalization of the classical scalar Jiles-Atherton model [6]. Specifically, in [5] the nonlinear problem is solved in the Fourier domain by means of Picard-Banach iterations whose convergence in a wide range of material properties can be analytically proved [7]. The numerical implementation has been carried out in a parallel environment, with a fast computation of the relevant matrix-by-vector products via a suitable sparsification of the full matrices involved [8]. In the full paper we extend the method to include an efficient numerical modeling of incremental permeability and harmonic analysis (in view of micro-magnetic characterization) and we present a detailed analysis of the computational aspects to prove the advantages of the proposed approach both in terms of memory allocation and CPU time. Several numerical results are discussed, showing the effectiveness of the proposed approach and its ability to solve problems of industrial relevance.

II. Numerical Formulation

We consider a set of conductors belonging to the conducting domain V_c, in the presence of magnetic materials belonging to the magnetic domain V_M. In the numerical model we embed an isotropic vector generalization [6] of the classical scalar Jiles-Atherton model which, despite the simplicity due to the local memory character, is one of the most widespread phenomenological hysteresis models.

The set of magneto-quasi-stationary Maxwell equations leads to the following weak form [9, 10]:

$$\int_{V_c} \mathbf{W}\cdot\eta\mathbf{J}dV + \frac{d}{dt}\int_{V_c}\mathbf{W}\cdot\left(\mathscr{A}\left[\mathbf{J},\mathbf{M}\right]+\mathbf{A}_s\right)dV + \sum_{h=1,N_E}\int_{S_h}\phi_h\mathbf{W}\cdot\hat{\mathbf{n}}dS = 0, \quad \forall\mathbf{W}\in S \quad \mathbf{J}\in S \tag{1}$$

$$\int_{V_M}\mathbf{W}_M\cdot\mathscr{G}^{-1}\left(\mathbf{M}\right)dV = \int_{V_M}\mathbf{W}_M\cdot\mathscr{B}\left[\mathbf{M},\mathbf{J}\right]dV, \quad \forall\mathbf{W}_M\in\mathbf{L}^2(V_M) \tag{2}$$

$S=\left\{\mathbf{J}\in H(div,V_C),\nabla\cdot\mathbf{J}=0 \text{ in } V_C, \mathbf{J}\cdot\hat{\mathbf{n}}=0 \text{ on } \partial V_C\setminus S_E\right\}$, the operators $\mathscr{A}$ and $\mathscr{B}$ give the magnetic vector potential and the magnetic flux density, respectively, for a given magnetization and prescribed currents and are defined through the Biot-Savart law. A set of N_E electrodes identified by the surface S_E may be present as a part of the boundary ∂V_c of the conducting domain. Each of these is characterized by the electric potential φ_h. We represent $\mathbf{J}$ as the linear combination of the basis functions $\mathbf{J}_j=\nabla\times\mathbf{T}_j\in S$, whereas the magnetization is expanded on piecewise constant functions [10]:

$$\mathbf{J}(\mathbf{r},t)=\sum_j I_j(t)\mathbf{J}_j(\mathbf{r})\,\mathrm{in}\,V_c\ ,\ \ \mathbf{M}(\mathbf{r},t)=\sum_j M_j(t)\mathbf{P}_j(\mathbf{r})\,\mathrm{in}\,V_M\ .$$

According to Galerkin's method we choose the $\mathbf{J}_k$'s in (1) and the $\mathbf{P}_k$'s in (2) as the weighting functions. Condition $\nabla\times\mathbf{T}_j\in S$ can be satisfied by adopting edge element shape functions [10] for $\mathbf{T}$, the uniqueness of the vector potential can be imposed as described in [10], [11]. The magnetic material is characterized by the constitutive equation $\mathbf{M}=\mathcal{G}(\mathbf{B})$ in V_M and the conducting material by $\mathbf{J=E}$ in V_c.

The solution of this time-periodic non-linear set of equations in the presence of hysteretic media cannot be obtained within a reasonable CPU time in the time-domain. For this reason, we use a frequency-domain iterative algorithm converging directly to the steady-state solution of the problem [3], which can be approximated by a truncated Fourier series:

$$I_j(t)=\mathrm{Re}\left\{\sum_{n=1}^{N_H}\tilde{I}_{j,n}e^{jn\omega_0 t}\right\} \tag{4}$$

where $\tilde{I}_{j,n}$ $(n=1,\ldots,N_H)$ is the phasor representing the n-th harmonics of I_j, ω_0 is the fundamental angular frequency and N_H is the number of harmonics in the numerical approximation. Similar expressions hold for the magnetization **M** and the magnetic induction **B**. Then, the problem is solved by means of the iterative fixed point procedure described in [5]. The iterative procedure requires the evaluation of the products of the full matrices corresponding to the discrete form of the linear operators $\mathcal{A}$ and $\mathcal{B}$, with the vectors representing the sources **J** and **M**. These products are efficiently evaluated by using the low rank property associated to the long range interactions as described in [7]. The products are implemented in an efficient parallel computational environment.

With respect to [5], which provides the basis for this work, the original contribution will be twofold: (i) a detailed analysis of the computational aspects in order to prove the advantages of the proposed approach both in terms of memory allocation and CPU time, and (ii) the numerical modeling of the measurement of incremental permeability and harmonic analysis. As mentioned in section I, the measurement of the incremental permeability $\Delta\mu=\Delta B/\Delta H$ and harmonic analysis play a key role in the micro-magnetic characterization of mechanical properties [1]. Incremental permeability can be measured by applying a magnetic induction to the specimen by means of an iron yoke supplied with a low frequency sinusoidal voltage establishing the working point along a main hysteresis loop. The field variation is obtained from the response of a small loop powered at a higher frequency and placed between the legs of the yoke. From the numerical point of view, the difficulty of analyzing this case lies in the need to model the response of a complex system produced by two driving currents having frequencies of different orders of magnitude. However, the field produced by the current at the higher frequency is weaker than the reference field at low frequency, so that we can decouple the two computations [12]. The details of the implementation will be discussed in the full paper.

III. Results and Discussion

The numerical model has already been validated by simple test cases [5]. Here we consider a system relevant to NDE testing configurations, similar to the one described in [5], composed of a linear magnetic yoke with two voltage-driven exciting coils and a ferromagnetic plate. The applied voltage is a sinusoid at f=200 Hz and N_H = 10. The model geometry is reported in [5] together with the Jiles-Atherton parameters. Two computations are presented: a scalar hysteretic case where the constitutive relation is hysteretic in the main direction of the field and is characterized by linear relationships in the two other directions and a second one where the fully hysteretic nature is taken into account. The different behaviour is particularly clear when the ferromagnetic plate is close to saturation. Figure 1 reports a snapshot of the magnetic induction field at time t=0s for the two cases.

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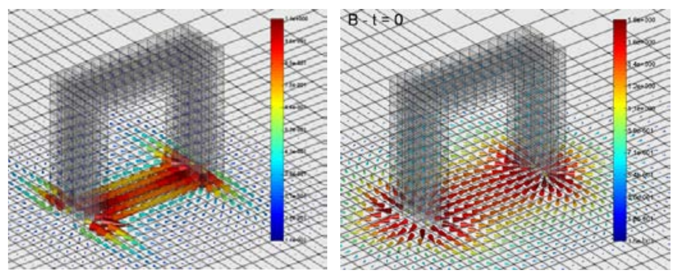

Fig. 1. Plot of **B** at t=0. Scalar (left) and vector (right) hysteresis model.

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