Equations of 3-D Electromagnetic Field with Direct Calculation of Flux and Eddy Current Densities

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Abstract—**The paper presents equations which allow us to calculate directly flux densities and eddy current densities in electromagnetic fields. A new method of the notation of the magnetic and eddy current field components and the connections of these components with space division networks is presented in this paper. Special attention is paid to taking into account the relations which associate flux densities and field intensities in individual parts of a ferromagnetic material.**

Index Terms—**Eddy currents, electromagnetic fields, electromagnetic modeling, magnetic flux density.**

I. Introduction

The Maxwell equations are the basis of almost all methods of electromagnetic field calculations. Examples and advantages of the use of the Maxwell equations in the integral form are presented in [1] – [4]. The proposed method of the formulation of the 3D electromagnetic field equations is similar to the reluctance network method [4], [5]. However, reluctances are not used in the creation of the magnetic field network. The determination of the reluctance values of a nonlinear magnetic circuit is quite inconvenient, especially if the hysteresis phenomenon is taken into account.

II. Division of Field Spaces

Field spaces are divided into elementary rectangular prism-shaped segments. The centres of segments are connected by branches. It is preferable to determine the so-called tree as a system of branches connecting the centres of all segments and not creating any closed path. One flux density component B_m and two field intensity components H_{mi} and H_{mo} are associated with only one of the branches which does not belong to the tree. The components H_{mi} are directed to the centres of the segments and the components H_{mo} are directed outside the segments. The B_p, H_{pi}, H_{po} components are associated with branches belonging to the tree. Four neighboring segments are shown in Fig. 1.

III. Equations of Magnetic Field

On the basis of the first Maxwell equation it is possible to formulate equations for independent meshes in the form

$$\sum_{k=1}^{8} a_k H_k = s_J J \tag{1}$$

where H_k is a field intensity component, a_k denotes a distance between the centre of the segment and the centre of the segment face, J is the density of the current of an external circuit or the density of an eddy current, and s_J denotes the area of the surface determined by a mesh.

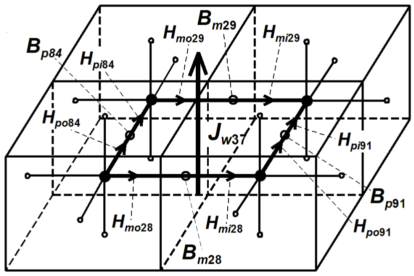

Fig. 1. Four neighboring segments of the magnetic field with a marked mesh; J_{w37} denotes a component of eddy current densities

All equations can be written as

$$\boldsymbol{A}_{mi}\boldsymbol{H}_{mi} + \boldsymbol{A}_{mo}\boldsymbol{H}_{mo} + \boldsymbol{A}_{pi}\boldsymbol{H}_{pi} + \boldsymbol{A}_{po}\boldsymbol{H}_{po} = \boldsymbol{S}_J\left(\boldsymbol{W}_{Jt}\boldsymbol{J}_t + \boldsymbol{W}_{Jw}\boldsymbol{J}_w + \boldsymbol{J}_{ex}\right) \tag{2}$$

where $\boldsymbol{H}_{mi}$, $\boldsymbol{H}_{mo}$, $\boldsymbol{H}_{pi}$ and $\boldsymbol{H}_{po}$ are the column vectors of the H_{mi}, H_{mo}, H_{pi}, and H_{po} components respectively, $\boldsymbol{A}_{mi}$, $\boldsymbol{A}_{mo}$, $\boldsymbol{A}_{pi}$ and $\boldsymbol{A}_{po}$ are matrixes of the distances a_k, $\boldsymbol{S}_J$ is the matrix of areas s_J, $\boldsymbol{J}_t$ and $\boldsymbol{J}_w$ are the column vectors and of the J_t, J_w components of eddy current densities, $\boldsymbol{W}_{Jt}$ and $\boldsymbol{W}_{Jw}$ are matrixes coupling individual meshes of the magnetic field network with appropriate J_t and J_w components of eddy current densities, $\boldsymbol{J}_{ex}$ is the vector of densities of external currents.

Using the Gauss law it is possible to write equations for the independent nodes of the network in the form

$$\sum_{k=1}^{ls} c_k B_k = 0 \text{ or } \Phi_{ex} \tag{3}$$

where B_k is a flux density component, c_k denotes the area of the segment face which is penetrated by the magnetic flux with the component B_k, and l_s is the number of the magnetic flux density components associated with the given node.
The product $c_k B_k$ represents a certain magnetic flux which flows into the segment or flows out of the segment. All equations in the form (3) can be written as follows

$$\boldsymbol{C}_m \boldsymbol{B}_m + \boldsymbol{C}_p \boldsymbol{B}_p = \boldsymbol{\Phi}_{ex} \tag{4}$$

where $\boldsymbol{B}_m$, $\boldsymbol{B}_p$ are column vectors of the B_m, B_p components respectively, $\boldsymbol{C}_m$, $\boldsymbol{C}_p$ are matrixes of the areas of segment faces. On the basis of (2) and (4) it is possible to formulate one equation in which the $\boldsymbol{B}_m$ column vector is unknown.

IV. Equations of Eddy Current Field

The network of the eddy current space is created similarly as previously, but only one component of the eddy current density is assigned to each branch. These components have the t subscript if they are associated with branches not belonging to the tree, otherwise the components have the *w* subscript. On the basis of the second Maxwell equation we can formulate equations for independent meshes in the form

$$\rho \sum_{k=1}^{4} d_k J_k = -s_B \frac{d}{dt} B \tag{5}$$

where ρ is the resistivity of a ferromagnetic material, d_k is the distance between the centres of segments, J_k denotes an eddy current density component, s_B denotes the area of the surface determined by a mesh, and B is the flux density of the magnetic flux which penetrates the surface area s_B.
All equations in the form (5) can be written in matrix form

$$\rho(\boldsymbol{D_t J_t} + \boldsymbol{D_w J_w}) = \boldsymbol{S_B}\left(\boldsymbol{W_{Bm}} \frac{d}{dt} \boldsymbol{B_m} + \boldsymbol{W_{Bp}} \frac{d}{dt} \boldsymbol{B_p}\right) \tag{6}$$

where $\boldsymbol{J_t}$, $\boldsymbol{J_w}$ are column vectors of the J_t, J_w components respectively, $\boldsymbol{D_t}$, $\boldsymbol{D_w}$ are matrixes of d_k distances, $\boldsymbol{S_B}$ denotes the matrix of areas s_B , $\boldsymbol{W_{Bm}}$ and $\boldsymbol{W_{Bp}}$. are the matrixes coupling the independent meshes of the eddy current network with the appropriate components B_m and B_p of the flux densities.

For each independent node of the eddy current network we can formulate equations which have the following form

$$\sum_{k=1}^{ls} l_k J_k = 0 \tag{7}$$

where l_k is the area of the segment face which is penetrated by the eddy current with the component J_k, and l_s denotes the number of eddy currents flowing into the given segment.
The product $l_k J_k$ represents a certain equivalent eddy current which flows into the segment or flows out of the segment. All equations in the form (7) can be written as follows

$$\boldsymbol{L_t J_t} + \boldsymbol{L_w J_w} = 0 \tag{8}$$

where $\boldsymbol{L_t}$, $\boldsymbol{L_w}$ are matrixes of the areas of segment faces.
On the basis of (6) and (8) it is possible to formulate one equation in which the $\boldsymbol{J_t}$ column vector is unknown.

V. Nonlinear materials

In many cases some segments represent a nonlinear ferromagnetic material. Then, any field intensity component can be shown in the form

$$H = gB + mf(B) \tag{9}$$

where $f(B)$ is a certain nonlinear function.
If a field intensity component H refers to a linear magnetic medium then the parameter g is equal to the reluctivity of this medium and the parameter m is equal to zero. Otherwise the parameters g are equal to zero and the parameters m are equal to one if the segments represent a nonlinear material. For all segments the relations between components can be written in the matrix form, which for the H_{mi} components is as follows

$$\boldsymbol{H_{mi}} = \boldsymbol{G_{mi} B_m} + \boldsymbol{M_{mi} F_{mi}}(\boldsymbol{B_m}, \boldsymbol{B_p}) \tag{10}$$

where the column vectors $\boldsymbol{F_{mi}}(\boldsymbol{B_m}, \boldsymbol{B_p})$ include the nonlinear functions $\boldsymbol{f}(B_m, B_p)$, which describe the relations between components in the segments of a nonlinear material.

The experimental verification was made for a ferromagnetic steel plate. A thin conductor was pulled through the centre of this material (Fig. 2). On the basis of the voltages of measurement coils the flux densities were calculated and compared with the calculated flux densities.

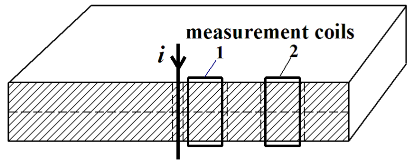

Fig. 2. The cross-section of a steel plate
with a current-carrying wire and measurement coils

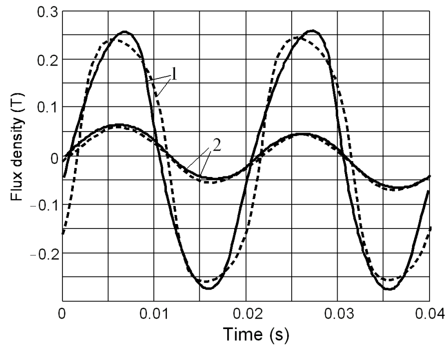

Fig. 3. Changes of flux densities; continuous line – the waveform determined on the basis of measured voltages, dashed line – the calculated waveform

VI. Conclusions

The presented method of the formulation of equations is a certain modification of the reluctance network method although reluctances are not used because it is quite difficult to determine them for non-linear materials. The proposed notation of the field components allows us to reduce the number of equations to the sum of the independent meshes of the magnetic and eddy current fields. By using these equations it is relatively easy to take into account nonlinearities of ferromagnetic material, especially the hysteresis phenomenon.

Acknowledgments

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References

[1] R. Albanese, G. Rubinacci, M. Canali, S. Stangherlin, A. Musolino and M. Raugi, "Analysis of a transient nonlinear 3-D eddy current problem with differential and integral methods," *IEEE Trans. on Magn.*. vol. 32, no. 3, pp. 776-779, 1996.

[2] M. Clemens and T. Weiland, "Discrete electromagnetics: Maxwell's equations tailored to numerical simulations," *Int. Compumag Society, Newsletter*, vol. 8, no. 2, pp. 13-20, 2001.

[3] N. Esposito, A. Musolino and M. Raugi, "Equivalent network model for magnetic field and circuital analysis of transformers including hysteresis effects," *IEEE Trans. on Magn*., vol. 32, no. 3, pp. 1094-1097, 1996.

[4] M. Fukuoka, K. Nakamura and O. Ichinokura, "Dynamic analysis of plenatary-type magnetic gear based on reluctance network analysis," *IEEE Trans. on Magn.*, vol. 47, no. 10, pp. 2414-2417, 2011.

[5] Sykulski, J. Stoll, R. Sikora, K. Pawluk, J. Turowski and K. Zakrzewski, *Computational Magnetics*, London, Chapman & Hall, 1995.

[6] J. Turowski, M. Turowski and M. Kopec, "Method of three-dimensional network solution of leakage field of three-phase transformers," *IEEE Trans. on Magn.*. vol. 26, no. 5, pp. 2911-2919, 1990.