Rigorous Capacitance-Extraction Method for Metamaterial Resonator Equivalent Circuits

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***Abstract*—In this paper, a computational methodology for the capacitance extraction of metamaterial resonators, like the edge-coupled split ring resonator (EC-SRR), is introduced. Initially, the new algorithm treats the EC-SRR as an open two-conductor system and determines its capacitance matrix via a static finite difference solver. This matrix is then utilized for the calculation of the operational capacitance between the two conductors. Finally, the desired capacitance of the quasi-static, equivalent EC-SRR circuit model is efficiently derived, taking into account the charge distribution on the structure at its first resonance frequency.**

***Index Terms*—Capacitance, computational electromagnetics, lumped circuits, metamaterials.**

I. Introduction

During the past decade, metamaterials, broadly defined as artificial composites with properties not encountered in natural media, have attracted the impressive interest of researchers in the area of electromagnetics [1]–[6]. Most metamaterials comprise resonant elements with their desired features usually occuring near their first resonance, which lies within the quasi-static limit. Therefore, circuit models with lumped *LC* elements have been proposed as a physically meaningful analysis and design tool. Typically, the values of these lumped elements are obtained through empirical formulas resulting from microwave circuit theory, or analytical techniques, in which the current distribution across the resonator is presumed to be known [1], [7]. However, the accuracy of these approaches has been questioned, due to their approximate assumptions.

To the best of our knowledge, computational methods have not been extensively employed on the problem of metamaterial circuit modeling. Actually, these algorithms can accurately extract lumped element values, without any extra conventions. To this aim, a novel numerical technique is presented in this paper for the precise and realistic extraction of the equivalent circuit capacitance of several metamaterial resonators, such as the edge-coupled split-ring resonator (EC-SRR). According to the key concept, the partial capacitances of the open two-conductor system are first computationally obtained from a static finite difference solver and afterwards the operational capacitance between these two conductors is evaluated via a robust electromagnetics approach. At a final stage, the desired equivalent circuit capacitance is retrieved by considering – unlike other schemes – the resonator's curved geometry and the incident field orientation. The proposed method is successfully applied to EC-SRRs with various dimension sets and the results are compared with those from existing microstrip capacitance approximation formulas to prove its advantages.

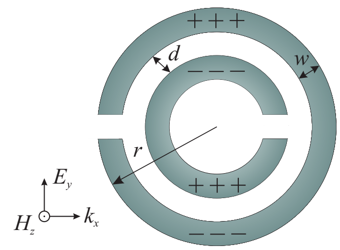

Figure 1. Depiction of the EC-SRR geometry, the incident wave polarization, and the charge distribution induced on the structure at its first resonance.

II. Development of the Extraction Methodology

The EC-SRR, illustrated in Fig. 1, is composed of two metallic circular strips. Since the capacitance matrix of an open system of conductors depends only on geometrical features, its partial capacitances can be acquired through static numerical analysis. For this purpose, the CST EM^{TM} Studio commercial package [8] has been selected for our simulations. Both rings are modeled as perfect electric conductors, yielding a lossless structure and a corresponding *LC* equivalent circuit at its first resonance. A potential $\phi_1 = 1\,\text{V}$ is applied on the inner ring, while a $\phi_2 = 0\,\text{V}$ is selected for the outer one. Next, and by an electrostatic formulation, we acquire the capacitance and induction coefficients c_{ii} and c_{ij} $(i \neq j)$, respectively, from which self-partial capacitances, C_{ii}, and mutual partial capacitances, C_{ij} $(i \neq j)$, can be evaluated through

$$Q_1 = C_{11}\phi_1 + C_{12}(\phi_1 - \phi_2), \tag{1}$$

$$Q_2 = C_{21}(\phi_2 - \phi_1) + C_{22}\phi_2, \tag{2}$$

with $C_{ii} = c_{i1}+c_{i2}+...+c_{in}$, $C_{ij} = -c_{ij} = -c_{ji}$ and Q the charges. Capacitance, C_o, between the conductors, for an open-ended system, is in fact their "equivalent" or "working" capacitance

$$C_o = C_{12} + \frac{C_{11}C_{22}}{C_{11} + C_{22}}. \tag{3}$$

However, before calculating the value of the resonator's lumped circuit capacitance, the charge distribution at the first resonance must be considered. Owing to the polarization of the incident electric field in Fig. 1, there are two distinct areas where voltages of opposite orientation occur, leading to the presence of two series capacitors in the equivalent circuit representation [1]. Bearing in mind the symmetry of the EC-SRR, the equivalent circuit capacitance is, thus,

$$C = \frac{(C_o/2)(C_o/2)}{C_o/2 + C_o/2} = \frac{C_o}{4}. \tag{4}$$

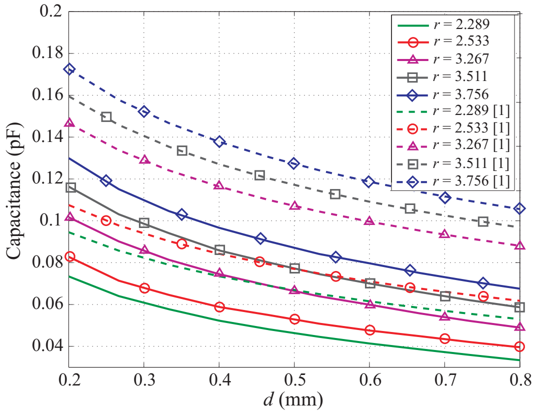

Figure 2. Equivalent circuit capacitance versus the strips spacing, d, for different external radii, r (in mm), and a fixed strip width $w = 0.5$ mm.

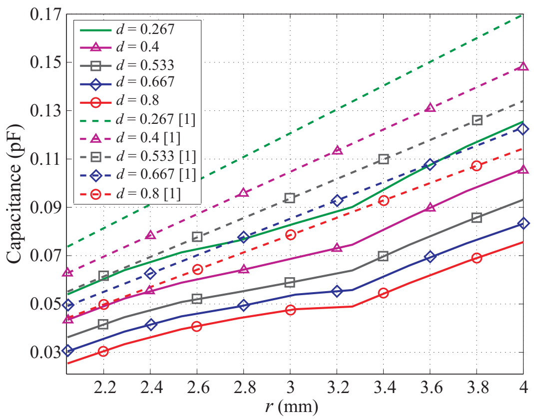

Figure 3. Equivalent circuit capacitance versus the external radius, r, for different strips spacings, d (in mm), and a fixed strip width $w = 0.5$ mm.

A significant asset of the new scheme is that (4) can be easily implemented together with the previous numerical analysis, to accurately determine the required capacitance of the equivalent lumped circuit of the EC-SRR, without any assumptions.

III. Numerical Results

To certify the merits of the proposed methodology, various EC-SRRs with different sets of external radii, r, spacings between the strips, d, and strip widths, w, are addressed. It is mentioned that other parameters of interest could be the width of the gaps, g, or the metal strip thickness, yet their effect on the first resonance characteristics is deemed negligible and hence not investigated herein. Moreover, all numerical results are presented in comparison with those derived through the expressions in [1]. The latter empirical formulas originate from microwave theory and were initially used for the calculation of the capacitance per unit length of parallel microstrip lines.

In this context, Fig. 2, shows the EC-SRR capacitance as a function of the spacing between the strips for various radii. As detected, both schemes are qualitatively similar, although our technique leads to smaller values than those of [1]. This is essentially attributed to the geometry of the EC-SRR rings, which more closely resemble to curved microstrip bends with lower capacitances, rather than to parallel microstrip lines, as assumed in [1]. This is, also, the case for the capacitance

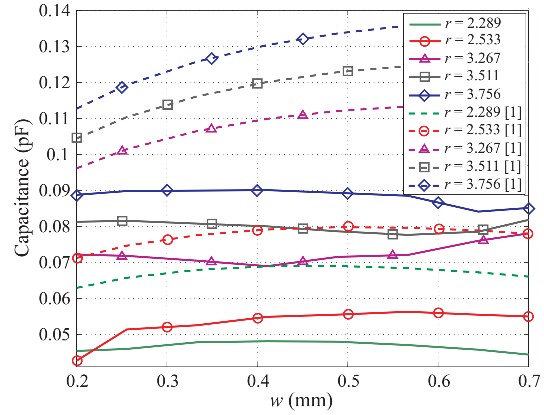

Figure 4. Equivalent circuit capacitance versus the strip width, w, for different external radii, r (in mm), and a fixed strips spacing $d = 0.5$ mm.

versus the external radius for different line spacings, depicted in Fig. 3. However, in Fig. 4, where the capacitance is a function of the strip width for different external radii, although both algorithms are qualitatively similar for small radii values, significant deviations are observed for larger radii. This fact can be appointed to the specific geometry, as well; indeed, it becomes apparent that the larger the radius, the less the influence of the strip width on the final capacitance value. The prior results reveal the realistic performance of our technique, since it considers the important issue of strip curvature.

IV. Conclusion

A generalized method for the numerical extraction of the equivalent lumped element circuit capacitance for diverse metamaterial resonators has been developed in this paper. The proposed technique has been applied to the EC-SRR for several sets of dimensions, and results have been compared with those obtained via empirical formulas, showing good qualitative agreement. Thus, it can be a trustworthy and accurate means for improved empirical formulas or the derivation of the proper correction factors for existing approaches.

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