A Hybrid Boundary Element Method-Reluctance Network Method for Open Boundary 3D Non Linear Problems

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Abstract—**A new method to solve magnetostatic field problems was presented recently. A scalar potential formulation coupling a hybrid Boundary Element Method and a Network Reluctance Method (BEM-NRM) was used in order to solve 2D problems. This paper presents the extension of this method for problems in the 3D nonlinear case. The approach will be compared with FEM in terms of computation time and accuracy. It will be shown that the approach could be efficiently used for the pre-sizing of actuators.**

***Index Terms*—Actuators, boundary element method, coupling of numerical techniques, magnetostatics, reluctance network method.**

I. Introduction

Based on the principle of equivalent magnetic circuits, Network Reluctance Method (NRM) is an approach enabling a very quick evaluation of electromagnetic quantities in devices [1]. Although easy to implement, NRM results can be somewhat inaccurate in a first step especially if significant flux leakages are associated to the device. Thus, the network can be improved by comparing the results with those from the FEM model and rebuilding a more representative NRM. Unfortunately, the development of such an approach can be very extensive.

Tools based on Finite Element Method (FEM) provide to the user effective representation of the device. Compared to NRM, the geometric and physical properties are both precisely and quickly described. On the one hand, the FEM can solve a wide range of problems; on the other hand, resolution times can be prohibitive for an actuators pre-sizing step. This question is more meaningful in a context of optimization with a large number of parameters.

A method combining fast computation time and quite good accuracy would be very appropriate during the pre-sizing process. This paper proposes a new hybrid method combining the Boundary Element Method (BEM), for the surrounding region (the air), and NRM, for magnetic regions in order to combine their respective advantages. This approach has been already presented for 2D applications [2]. The novelty here is its extension to 3D problems.

II. Boundary Element Method (BEM) for air

Let us consider the third Green's identity applied in the air region to the scalar magnetic potential ϕ . We have:

$$c(\mathbf{x_0})\phi(\mathbf{x_0}) = \int_{C_e}\left(\phi\frac{\partial G}{\partial \mathbf{n}} - G\frac{\partial \phi}{\partial \mathbf{n}}\right)dC\,, \qquad (1)$$

where $\phi(\mathbf{x_0})$ represents the magnetic scalar potential at $\mathbf{x_0}$ point, G is the 1/r Green's function, C_e is the boundary of magnetic regions, $\mathbf{n}$ its external normal and c is the solid angle subtend by the boundary C_e.

We consider a constant distribution for the potential and its normal derivative on the border (0-order shape function). Thanks to a point matching approach at centroid of each element, c_i is identically equal to 1/2 and we get a very simple discretized expression of (1) in matrix forme:

$$\mathbf{H}\,\mathbf{U_{BEM}} + \mathbf{T}\mathbf{Q_{BEM}} = 0\,, \qquad (2)$$

where $\mathbf{U_{BEM}}$ and $\mathbf{Q_{BEM}}$ are vectors of dimension N and $\mathbf{T}$ and $\mathbf{H}$ matrices are associated to following expressions, [3]:

$$T_{ij} = \int_{C_j} G_i\, dC\,, \qquad H_{ij} = c_{ij} - \int_{C_j}\frac{\partial G_i}{\partial \mathbf{n}}dC\,, \qquad (3)$$

where c_{ij} is null if i≠j and equals 1/2 if i=j. T is an invertible matrix, thus we can write a relation linking the flux to the potential on the boundary:

$$\mathbf{Q_{BEM}} = \mathbf{P_{BEM}}\mathbf{U_{BEM}}\,, \qquad (4)$$

$$\mathbf{P_{BEM}} = -\mathrm{inv}(\mathbf{T})\mathbf{H}\,. \qquad (5)$$

Let us notice that the matrix $\mathbf{P_{BEM}}$ is a fully populated matrix.

III. Reluctance Network Method (RNM) for magnetic material

We decompose the magnetic domain into bricks and we introduce a reluctance network inside each of them. This network can optionally contain sources of magnetomotive force F_{kt}.

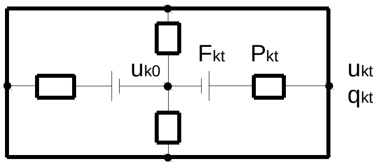

Fig. 1. Introducing NRM in the brick of the magnetic domain

Let us suppose a magnetic region Ω_m , discretized by K bricks. Each brick k contains T flux tubes connecting the central potential u_{k0} to the potentials of facets u_{kt}. q_{kt} is the flow through the facet t of the brick k. The following relation can be written:

$$q_{kt} = (u_{kt} - u_{k0} + F_{kt})P_{kt}\ , \quad (6)$$

where P_{kt} is the permeance of a flux tube and can be calculated from its permeability μ_{kt}, its length L_{kt} and its cross section S_{kt}. By isolating the central potential in the equation and considering the flux conservation, we get for each face number kt:

$$q_{kt} = \left[\left(u_{kt} - \frac{\sum_{t=1}^{T}(u_{kt})P_{kt}}{\sum_{t=1}^{T}P_{kt}}\right) + \left(F_{kt} + \frac{\sum_{t=1}^{T}(F_{kt})P_{kt}}{\sum_{t=1}^{T}P_{kt}}\right)\right]P_{kt}\ . \quad (7)$$

Magnetomotive force F_{kt} can be computed according to Ampere's law by assuming a uniform inductor field into a subset of the domain bricks. More details about the technique will be provided in the full paper.

IV. Coupling both Methods

It remains now to couple both methods. Equation (7) can be rewritten as follows, [2]:

$$\mathbf{Q}_{\mathbf{NRMborder}} = \mathbf{P}_{\mathbf{NRM}}\mathbf{U}_{\mathbf{NRM}} + \mathbf{F}_{\mathbf{NRM}}\ , \quad (8)$$

where $\mathbf{P}_{\mathbf{NRM}}$ represents the permeance matrix, $\mathbf{U}_{\mathbf{NRM}}$ the potentials, $\mathbf{Q}_{\mathbf{NRMborder}}$ the flux flowing through the border of the device, and $\mathbf{F}_{\mathbf{NRM}}$ the magnetomotive forces created by sources. Let us notice that $\mathbf{P}_{\mathbf{NRM}}$ is a sparse matrix and have more unknowns than equation.

Thanks to (4) and (8), we can easily eliminate external flux unknowns and build a matrix representing the BEM-NRM coupling:

$$\mathbf{P}_{\mathbf{BEM-NRM}}\mathbf{U}_{\mathbf{NRM}} = \mathbf{F}_{\mathbf{NRM}}\ , \quad (9)$$

where $\mathbf{P}_{\mathbf{NEM\text{-}NRM}}$ is built using $\mathbf{P}_{\mathbf{BEM}}$ and $\mathbf{P}_{\mathbf{NRM}}$ matrix.

The nonlinear ferromagnetic material is represented by an initial permeability and a saturation level. We use the fixed point method to solve the nonlinear system. Once the problem has been solved, the Maxwell stress tensor approach is used to compute forces.

V. Results and Comparisons

In this section, we present the comparisons between a FEM model and the BEM-NRM model of an actuator represented by Fig. 2.

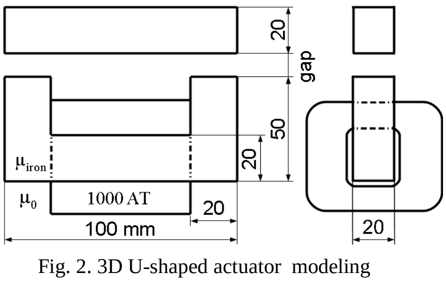

Fig. 2. 3D U-shaped actuator modeling

In order to present the comparison, we have to consider the variation of the air gap thickness. Fig. 2 presents the force errors computed with BEM-NRM as a function of the air gap thickness in comparison with a FEM reference computed by Flux® [4], with a very dense mesh. FEM 2, FEM 3 and FEM 4 represent others FEM solutions using different mesh size.

The idea is to study the evolution of the accuracy and the computation time. The error is divided by a reference value of force in order to get a relative one.

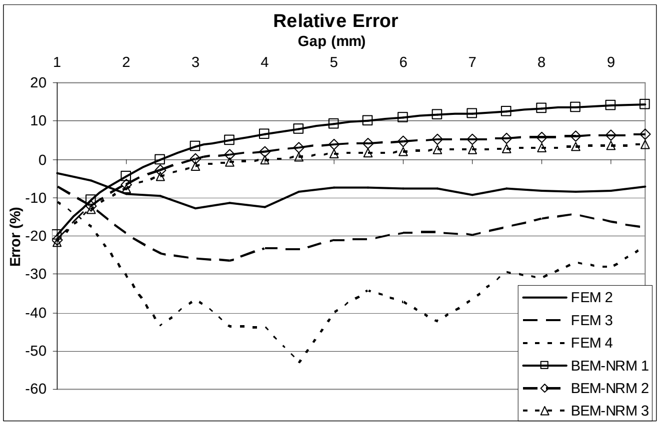

Fig. 3. Relative error as a function of gap

Twenty different positions are computed for each problem. Computation times and number of degrees of freedom are presented on the table I.

TABLE I

COMPUTATION TIME

Method	FEM 2	FEM 3	FEM 4	BEM-NRM 1	BEM-NRM 2	BEM-NRM 3
Degrees of freedom	27687	9372	4522	52	328	1008
Time (s)	770	285	97	1	28	395

It is shown that with FEM we can not decrease drastically the mesh size. Otherwise, the quality of the result becomes poor (see results of FEM 4).

Inaccuracies for the BEM-NRM appear when the gap becomes too small. It can be certainly explained by the use of zero order shape functions. However, our new method remains very efficient if we consider its speed comparing to the quality of its results (see results of BEM-NRM 1 associated to 52 degrees of freedom).

VI. Conclusion

With our new method a very low number of elements and a very low computation time are needed to get an accurate result. Getting an equivalent accuracy with a FEM necessitates much more time.

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