Particulate model for magnetic field and force computation

Z. László and J. Füzi
Wigner RCP Solid State Physics Institute
Konkoly-Thege Miklós út, 29-33, H-1121 Budapest, Hungary
laszlo.zoltan@wigner.mta.hu

***Abstract*—A particulate model, based on micromagnetic analogy is proposed for magnetic field computation in ferromagnetic media. The force acting on a magnetized body in magnetic field is computed by summation of the contributions of forces acting on dipoles associated with small volume elements of the body. The effect of the volume elements size and aspect ratio is investigated and the accuracy of computations verified against measurement.**

***Index Terms*— magnetic moments, magnetic forces, magnetic levitation, permanent magnets.**

I. Introduction

There exist several models to describe the magnetic characteristic of ferromagnetic materials, including non-linearity and hysteresis. One possibility is to compute the behavior of magnetic dipole assemblies using zero temperature Monte-Carlo simulation in order to reproduce vector hysteresis characteristics. Dipole-dipole and exchange interactions are considered. Shape anisotropy of the clusters model anisotropic features. The obtained model can then be included in magnetostatic computation codes, based e.g. on the volume element method. Similarly, the force acting on a ferromagnetic body can be computed by dividing it into small volume elements and considering the interaction between the external field and the magnetic moment of each element. Simple cases of one degree of freedom levitation between permanent magnets have been constructed and the repelling forces between magnets measured as a function of the distance between their surfaces. The results are applied to verify the robustness and accuracy of the proposed force computation method.

II. Particulate Vector Hysteresis Model

The magnetic field sensed by a particle of a dipole cluster is given in equation 1, where $\boldsymbol{H}_l$ is the field strength at the location of particle l, consisting of the externally applied field ($\boldsymbol{H}_{\text{ext}}$) and the field created by the other members of the cluster; $\boldsymbol{m}_k$ is the magnetic dipole moment of particle k and $\boldsymbol{r}_{kl}$ the relative position vector of particles k and l; J_{lk} is the exchange coefficient, different from zero only if dipoles l and k are in first (or first few) neighbor locations.

$$\boldsymbol{H}_l = \boldsymbol{H}_{ext} + \sum_{\substack{k=1\\k\neq l}}^{N} J_{lk}\boldsymbol{m}_{kl} - \frac{1}{4\pi}\sum_{\substack{k=1\\k\neq l}}^{N}\left(\frac{\boldsymbol{m}_k}{\boldsymbol{r}_{kl}^3} - 3\frac{(\boldsymbol{m}_k\cdot\boldsymbol{r}_{kl})\boldsymbol{r}_{kl}}{\boldsymbol{r}_{kl}^5}\right) \quad (1)$$

$$k = 1, N$$

The orientation of each particle is determined to minimize the angle between the field vector at the particle location and its orientation [1].

To enhance convergence, the orientations of the magnetic moments are confined to several fixed directions.

The magnetic moments of randomly selected particles are oriented parallel to the allowed direction closest to that of the field created by all the other particles, added to the given external field. This process is repeated until no change is encountered (all moments find their equilibrium orientation). The average magnetization is the vector sum of individual moments divided by the volume occupied by the cluster.

To create a model applicable in field computations, it is desirable to have a reduced number of dipoles in order to reduce the required number of operations. A possibility to achieve this is to apply a "virtual infinite extension" in one direction, by adding the field created by two virtual neighbor clusters in the selected direction, the corresponding dipoles having identical orientations to those in the mother cluster.

The hysteresis characteristics of a rod-shaped cluster, with simple cubic lattice, aspect ratio 1:3:1 (63 dipoles), 25% of the dipoles removed (leaving 47 dipoles in the cluster), virtually extended in the (1,0,0) direction, is plotted in figure 1.

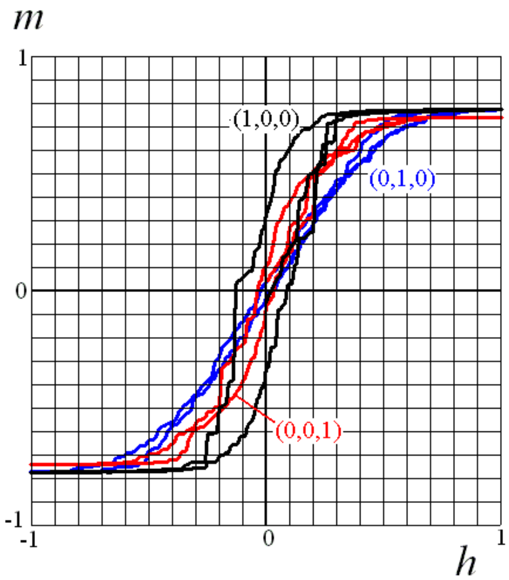

Fig. 1. Magnetizing characteristics of virtually extended rod-shaped dipole cluster of simple cubic structure, with 25% of the dipoles removed from random locations.

With the hysteresis model fitted to the investigated ferro-magnetic material, the integral equation method can be applied for computation of magnetic fields in large areas, in the presence of ferromagnetic bodies. Only the bodies are divided into uniformly magnetized elements and coefficient matrices constructed, which give the influence of the element magnetizations on each other:

$$\boldsymbol{H}_k = \boldsymbol{H}_0 + \sum_{l=1}^{N} [\mathbf{C}_{kl}]\boldsymbol{M}_l \quad ; \quad k = 1, N \tag{2}$$

Then the magnetizations of the elements are computed iteratively, so that the ***M*-*H*** relationship on each element fulfills the magnetic characteristic of the material. As for the iterative method, a fixed-point technique can be applied based on the nonlinear minimization of an error function:

$$\varepsilon = \sum_{k=1}^{N} \left| \boldsymbol{M}_k - \tilde{\boldsymbol{M}}_k \right|^2 \rightarrow \min \quad \tilde{\boldsymbol{M}}_k = \mathbf{H}(\boldsymbol{H}_k) \ . \tag{3}$$

III. Magnetic Force Computation

The force acting between two NdFeB permanent magnets grade N35 is computed as well as measured. The experimental setup eliminates all degrees of freedom except one. Friction between the moving magnet and the mechanical constraints is a source of systematic error. The magnetic force is determined in the following way [2]: both magnets are divided into small volume elements and dipoles residing at the center of the element are considered with moment equal to the magnetization times the volume of the element. The forces between dipole pairs, one from each two magnet are computed and summed.

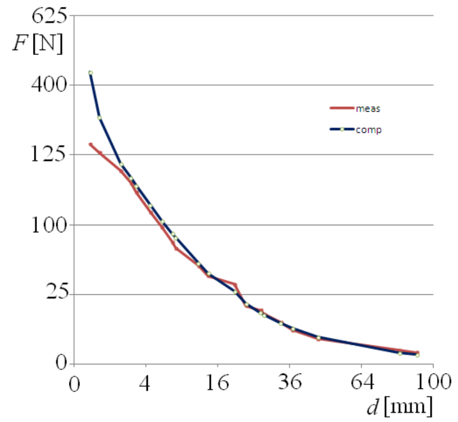

Fig. 2. Force in vertical direction as a function of gap of a pair of magnets length x width x height: 50 mm x 20 mm x 10 mm, magnetized along height

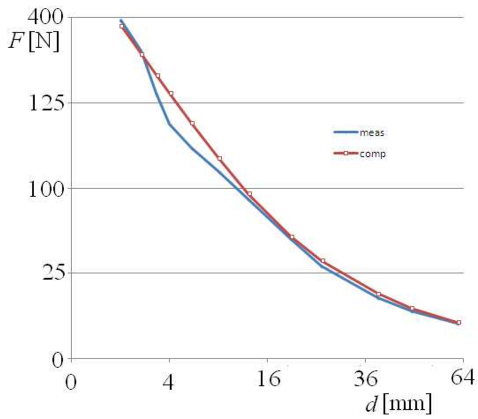

Fig. 3. Force in vertical direction as a function of gap of a pair of magnets length x width x height: 50 mm x 20 mm x 20 mm, magnetized along height

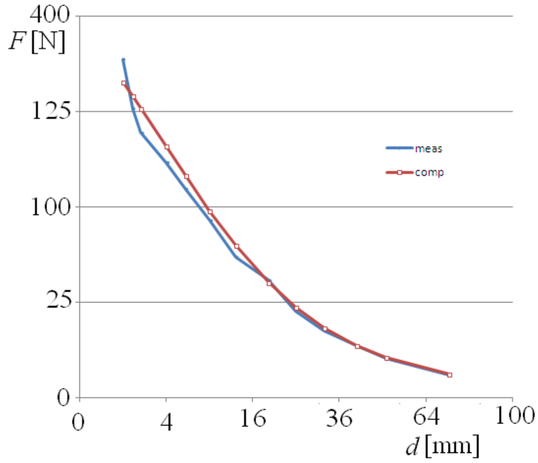

Fig. 4. Force in vertical direction as a function of gap of a pair of magnets length x width x height: 50 mm x 20 mm x 10 mm, magnetized along width

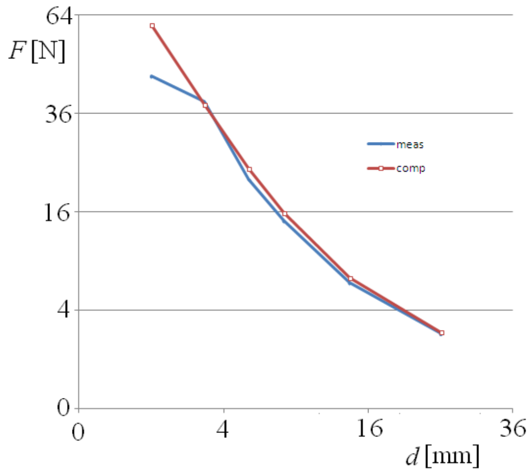

Fig. 5. Force in vertical direction as a function of gap of a pair of magnets length x width x height: 10 mm x 20 mm x 50 mm, magnetized along length

The measured and computed force as a function of gap between a pair of permanent magnets in various arrangements are plotted in figures 2, 3, 4 and 5. The adopted volume element size is roughly cubic, of 1.5 mm edge length.

The results yielded that the computational error can be kept reasonably low if the volume elements are small compared to the distance between them. When this is not possible – pairs of volume elements close to the common surface of touching magnets – more accurate computation of the field of the source magnet is necessary [3] and care has to be taken that the element size in the other magnet is not large compared to the variation of the source field gradient.

References

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