# Numerical Modelling of Axissymetrical Ferrite-Core Probes over Planar Specimens Using a Coupled FIT/Semi-Analytical Formulation

Audrey Vigneron, Anastassios Skarlatos and Christophe Reboud
CEA, LIST,
Centre de Saclay, F-91191 Gif-sur-Yvette cedex, France
anastasios.skarlatos@cea.fr

***Abstract*—A hybrid approach coupling the finite integration technique (FIT) with a semi-analytical model is presented for the calculation of the eddy-current interaction between axis-symmetrical ferrite-core probes and planar multi-layered conducting and/or ferromagnetic specimens. The initial 3D problem is decomposed in this way into two simpler ones, each one being symmetrical in its proper frame of reference. The corresponding exploitation of the symmetries of the involved objects in combination with a modal based formulation of the classical FIT discretization scheme allows us to avoid the 3D meshing of the probe-specimen ensemble and hence to reduce the size of the numerical problem to be solved.**

***Index Terms*—Nondestructive testing, magnetic cores, numerical simulation.**

## I. Introduction

Ferrite elements are widely used in nondestructive testing (NDT) applications. Industrial eddy-current probes are usually equipped with ferrite cores of high relative magnetic permeability in order to maximise the magnetic flux intercepted by the probe. Iron cores are involved in electromagnets used for the creation of strong static magnetic fields in electromagnetic acoustic transducers (EMATs). There is therefore strong interest in modelling the interaction of such elements with the inspected specimen.

From the practical point of view, we are primarily interested in the modelling of rotationally symmetric ferrite/magnetic elements, since these are the forms involved in the vast majority of NDT applications. The interaction of these elements with planar work-pieces is a two-dimensional problem so long their axis is normal to the specimen interface. Nevertheless, deviations from the nominal probe orientation (i.e. normal to the specimen interface) result in symmetry breaking, and thus lead to a three-dimensional problem.

The general three-dimensional problem of the eddy-current interaction between a ferrite-cored probe and a conducting and/or magnetic specimen can be addressed using 3D numerical tools like the finite element method (FEM) [1], [2]. Recalling, however, that the total arrangement is composed by elements, which in their proper frame of reference are symmetrical, 3D meshing can be avoided if each object is considered separately, and their interactions are then combined in the framework of a coupled formulation.

In this work the coil-core ensemble is discretized and treated using the finite integration technique (FIT) [3], [4], whereas the effect of the planar specimen is taken into account by means of a semi-analytical modal approach [5]. The transition from the one subproblem to the other is realized on the basis of the equivalence theorem and using an iterative two-step procedure [6].

Once the probe-specimen interaction has been calculated, the evaluated electromagnetic field inside the latter can be provided to a fast integral-based solver, in order to evaluate cracks signals detected by a probe during the specimen inspection [7].

## II. Mathematical Formulation

Let us consider a cylindrical coil supplied with a lossless, rotationally symmetric, ferrite core. The coil with its core is located over a planar green stratified specimen, made of conducting and possibly ferromagnetic layers. Insulating sheets or air gaps may also be included to separate some layers, prohibiting their galvanic contact. The orientation of the coil-core ensemble in respect to the specimen is arbitrary. The considered configuration is depicted in Fig. 1.

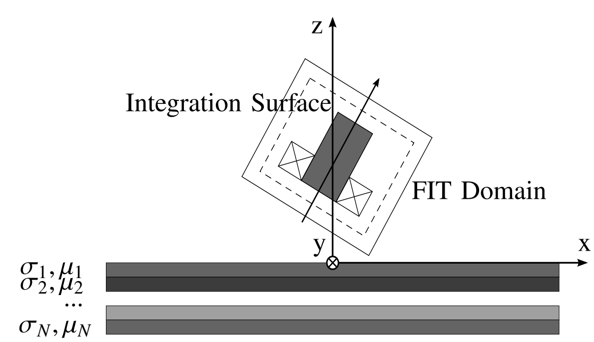


Figure 1: Considered configuration of a tilted ferrite-cored coil inspecting a planar multilayer specimen.

In a first step, the probe is enclosed into a fictitious coaxial cylindrical box, as shown in Fig. 1, in which a 2D FIT mesh is applied. Given that no conductive material is included inside the FIT domain and assuming that the applied frequency is sufficiently low, in order for the quasistatic approximation to be valid, the corresponding electromagnetic problem reduces to the magnetostatic formulation. Let $\Phi$ be the discrete magnetostatic scalar potential allocated at the nodes of the primary grid. The FIT governing equation for the magnetostatic scalar potential reads [4]:

$$\widetilde{\mathbf{S}}\mathbf{M}_\mu\widetilde{\mathbf{S}}^T\Phi = -\widetilde{\mathbf{S}}\mathbf{M}_\mu\widehat{\mathbf{h}}^{(i)} \tag{1}$$

where $\widetilde{\mathbf{S}}$ is the discrete div operator, $\mathbf{M}_\mu$ the material matrix for the magnetic permeability and $\widehat{\mathbf{h}}^{(i)}$ stand for the magnetic grid-voltage vector, which represents a partial solution to the Ampère's grid equation $\mathbf{C}\widehat{\mathbf{h}}^{(i)} = \widehat{\widehat{\mathbf{j}}}$.

In case of a generalised Neumann boundary condition (i.e. the normal derivative of the scalar potential on the boundary takes a non-zero, fixed value), the right hand side of (1) must be complemented with a vector containing their given values $\widehat{\widehat{\mathbf{b}}}^{(b)}$. This is equivalent with imposing a magnetic charge distribution upon the boundary.

The invariance of the geometry around the axis of revolution allows us to express the solution in terms of a Fourier series $e^{jm\phi}$, thus (1) reduces to a 2D matrix equation for each m:

$$\left([\widetilde{\mathbf{S}}\mathbf{M}_\mu\widetilde{\mathbf{S}}^T]_{\rho,z} - m^2[\mathbf{M}_\mu]_\phi\right)\Phi_m = -\widetilde{\mathbf{S}}\mathbf{M}_\mu\widehat{\mathbf{h}}^{(i)}\delta_{m0} + \widehat{\widehat{\mathbf{b}}}^{(b)}_m. \tag{2}$$

Notice that $\widetilde{\mathbf{S}}\mathbf{M}_\mu\widehat{\mathbf{h}}^{(i)}$ appears only for $m = 0$ ($\delta_{m0}$ stands for the Kronecker's delta) since the current distribution is rotationally symmetric by hypothesis. The impact of the specimen on the solution is taken into account via the equivalent source $\widehat{\widehat{\mathbf{b}}}^{(b)}$ on the FIT boundary.

Considering a closed surface inside the FIT domain, which encloses entirely the probe structure, the field at any point of the external to this surface space can be expressed via a superposition of Biot-Savart and Coulomb integrals, which reproduce the contributions from the tangential and normal to the integration surface field components, respectively:

$$\mathbf{B}^{(inc)}(\mathbf{r}) = \oint_{\partial V} \frac{\mathbf{B}(\mathbf{r}')[\mathbf{n}\cdot\mathbf{R}] - \mathbf{n}[\mathbf{B}(\mathbf{r}')\cdot\mathbf{R}] + [\mathbf{n}\cdot\mathbf{B}(\mathbf{r}')]\mathbf{R}}{4\pi R^3}\, dS' \tag{3}$$

where $\mathbf{n}$ is the outward pointing, normal unit vector and $\mathbf{R} = |\mathbf{r} - \mathbf{r}'|$.

The field perturbation due to the presence of the specimen can be obtained by the following Fourier integral [5]:

$$\mathbf{B}^{(ref)}(\mathbf{r}) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\tilde{B}_z^{(inc)}(u,v)}{a}\frac{a\mu_r - a_1}{a\mu_r + a_1}e^{-az} \times (iu\mathbf{e}_x + iv\mathbf{e}_y - a\mathbf{e}_z)e^{iux}e^{ivy}\,du\,dv \tag{4}$$

where $\tilde{B}_z^{(inc)}$ is the 2D Fourier transform of the normal to $z = 0$ plane incident magnetic induction, calculated by (3), and $a = \sqrt{u^2 + v^2}$.

The unknown normal component of the magnetic flux density on the FIT boundary is thus given by the superposition of the incident (direct) and the reflected field, namely

$$\widehat{\widehat{\mathbf{b}}}^{(b)}_{m,i} = (\mathbf{B}^{(inc)} + \mathbf{B}^{(ref)})\cdot\mathbf{n}\,\Delta A_i \tag{5}$$

where the normal magnetic flux component $i$ is multiplied with the area of the boundary facet $\Delta A_i$ in order to obtain the flux element $\widehat{\widehat{\mathbf{b}}}^{(b)}_{m,i}$ according to the FIT conventions. Equations (2) - (5), together with (3),(4), form a system of coupled equations, whose solution provides the magnetic field at every point of the problem domain. The solution of the above system is carried out iteratively via a two-step procedure [6].

## III. Results

As a first comparison, the coupled formulation has been tested for the case of a non-tilted coil with a cylindrical core over a conducting half-space. The inner and outer radius of the coil was 6 mm and 8 mm, respectively, and its length was 2 mm. The core radius was identical with the coil inner radius and its length was 8 mm. The relative magnetic permeability of the core was taken equal to 100 and its electrical conductivity was assumed to be negligible. The conductivity of the half-space was set to 1 MS/m. In Fig. 2 the results of the coupled formulation are in agreement with the numerical solution of the complete structure using the classic FIT formulation at 10 kHz and for a lift-off equal to 5.0 mm. Results corresponding to cases, where the coil is tilted, will be presented at the Conference.

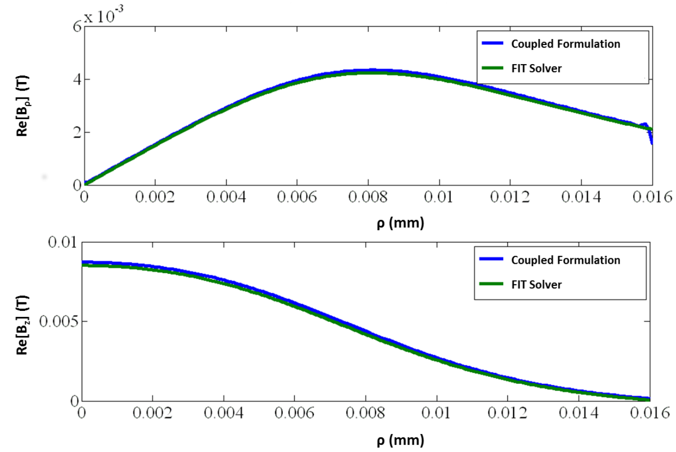


Figure 2: Tangential and normal component of the magnetic flux density on a parallel to the specimen plane above the coil.

## References


[1] Z. Zeng, L. Udpa, and S. S. Udpa, "Finite-element model for simulation of ferrite-core eddy-current probe," *IEEE Trans. Magn.*, vol. 46, no. 3, pp. 905–909, Mar. 2010.

[2] L. Huang, R. He, and Z. Zeng, "An extended iterative finite element model for simulating eddy current testing of aircraft skin structure," *IEEE Trans. Magn.*, vol. 48, no. 7, pp. 2161–2165, Jul. 2012.

[3] T. Weiland, "Time domain electromagnetic field computation with finite difference methods," *Int. J. Numer. Modell.*, vol. 9, no. 4, pp. 295–319, 1996.

[4] M. Clemens and T. Weiland, "Discrete electromagnetism with the finite integration technique," *Progress Electromagn. Res. (PIER)*, vol. 32, pp. 65–87, 2001.

[5] T. Theodoulidis, "Analytical model for tilted coils in eddy-current nondestructive inspection," *IEEE Trans. Magn.*, vol. 41, no. 9, pp. 2447–2454, Sep. 2005.

[6] A. Skarlatos, R. Schuhmann, and T. Weiland, "Solution of radiation and scattering problems in complex environments using a hybrid finite integration technique - uniform theory of diffraction approach," *IEEE Trans. Antennas Propag.*, vol. 53, no. 10, pp. 3347–3357, Apr. 2005.

[7] R. Miorelli, C. Reboud, D. Lesselier, and T. Theodoulidis, "Eddy current modeling of narrow cracks in planar-layered metal structures," *IEEE Trans. Magn.*, vol. 48, no. 10, pp. 2551–2559, Oct. 2012.