2D Magnetostatic Finite Element Simulation for Devices With Radial Symmetry

Dries Vanoost[1,2], Herbert De Gersem[2,3], Joan Peuteman[1], Georges Gielen[4], Davy Pissoort[1,4]
[1]Katholieke Hogeschool Brugge Oostende, Research Center Reliability in Mechatronics and ICT
[2]KU Leuven Kulak, Wave Propagation and Signal Processing Research Group
[3]KU Leuven, Department of Physics and Astronomy, Laboratory of Solid-State Physics and Magnetism,
[4]KU Leuven, Department of Electrical Engineering, Microelectronics and Sensors
dries.vanoost@khbo.be; herbert.degersem@kuleuven-kulak.be

***Abstract*—This paper proposes a 2D magnetostatic finite-element solver for radially symmetric devices, in analogy to 2D cartesian and axisymmetric solvers. Dedicated edge shape functions are developed and validated with regard to partity-of-unity, consistency and convergence properties. Peculiarities of radially symmetric modelling are discussed. Results obtained with the new solver are compared to 3D simulation results for a twin-rotor axial-flux permanent-magnet synchronous machine.**

***Index Terms*—Finite element methods, magnetostatics, convergence of numerical methods, partial differential equations, permanent magnet machines.**

I. Introduction

The standard 3D magnetostatic formulation reads

$$\nabla \times \left(\nu \nabla \times \vec{A}\right) = \vec{J}_\mathrm{s} - \nabla \times \vec{H}_\mathrm{s} \qquad (1)$$

with $\vec{A}$ the magnetic vector potential, ν the reluctivity, $\vec{J}_\mathrm{s}$ the applied current density and $\vec{H}_\mathrm{s}$ the source magnetic field strength of permanent-magnet (PM) material. When both the geometry and the excitations feature a translatory or cylindrical symmetry, (1) is typically solved in a 2D setting. Then, $\vec{J}_\mathrm{s}$ and $\vec{A}$ have only z- or θ-components respectively, whereas $\vec{H}_\mathrm{s}$ and the magnetic flux density $\vec{B} = \nabla \times \vec{A}$ are confined to the perpendicular xy- or rz-plane. For the translatory case, degrees of freedom (DoFs) are defined for the z-component of $\vec{A}$. The choice of DoFs for the cylindrical case is less obvious and was an item of discussion during the early nineties [1]. Eventually, a consensus arose on defining DoFs for $2\pi r A_\theta$ where A_θ is the θ-component of $\vec{A}$ [2]. 2D cartesian and axisymmetric solvers are standard, both in commercial software and freeware. Reductions to 2D for arbitary symmetries have been proposed in e.g. [3]–[5]. To the best of our knowledge, a 2D reduction of (1) for radial symmetry has not been proposed so far. This is remarkable because applications with a more or less radial symmetry exist (disk motors, cylindrical magnetic brakes) and an appropriate 2D reduction seems obvious. A workaround reported in [6] consists of stacking and coupling of number of thin cylindrical shells, each represented by a 2D cartesian model. This paper develops a 2D radially symmetric finite-element (FE) solver for the magnetic vector potential formulation and discusses some particularities which are not encountered in the cartesian and axisymmmetric cases.

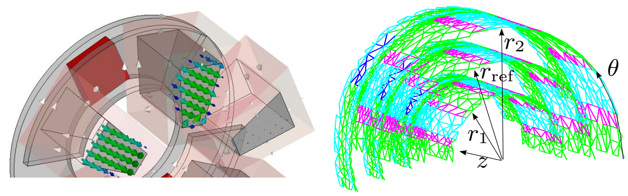

Fig. 1. Reduction of a disk motor to a cylindrical 2D shell.

II. 2D Reduction for Radial Symmetry

A cylindrical coordinate system (r, θ, z) with r the radial, θ the azimuthal and z the axial coordinate is considered (Fig. 1). The model reaches between $r = r_1$ and $r = r_2$ and has an arbitrary shape in the θz-plane. The model domain has a reluctivity ν, an applied current density $\vec{J}_\mathrm{s} = (h_\mathrm{j}/r, 0, 0)$ and a source magnetic field strength $\vec{H}_\mathrm{s} = (0, h_\theta/r, h_z)$ where ν, h_j, h_θ and h_z only depend on θ and z. The specific dependencies of $\vec{J}_\mathrm{s}$ and $\vec{H}_\mathrm{s}$ on r ensure the divergence-freeness of the current density and the curl-freeness of the coercitivity. A reference plane Γ_ref at a reference radius r_ref is considered. For modeling convenience and visualisation, the planar coordinates $(r_\mathrm{ref}\theta, z)$ are used. The magnetostatic formulation (1) in terms of $\vec{A} = (A_r, 0, 0)$ becomes

$$-\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\nu}{r}\frac{\partial A_r}{\partial\theta}\right) - \frac{\partial}{\partial z}\left(\nu\frac{\partial A_r}{\partial z}\right) = \frac{1}{r}\left(h_\mathrm{j} - \frac{\partial h_z}{\partial\theta} + \frac{\partial h_\theta}{\partial z}\right) \qquad (2)$$

$$\frac{\partial}{\partial r}\left(\nu\frac{1}{r}\frac{\partial A_r}{\partial\theta}\right) = 0 \quad ; \qquad (3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\nu\frac{\partial A_r}{\partial z}\right) = 0 \quad . \qquad (4)$$

The additional equations (3) and (4) force the solution $A_r(r, \theta, z)$ to have a particular dependence on the radial coordinate r, i.e.,

$$A_r = r f(\theta) + \frac{1}{r} g(z) \quad , \qquad (5)$$

where $f(\theta)$ only depends on θ and $g(z)$ only depends on z. The magnetic flux density is then $\vec{B} = (0, \frac{1}{r}\frac{\partial g}{\partial z}, -\frac{\partial f}{\partial\theta})$. Because the azimuthal and axial components of $\vec{B}$ have different dependences on r, the reduced formulation is not capable of representing a bended magnetic field. Nevertheless,

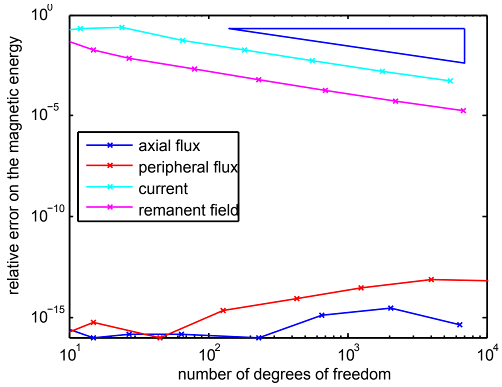

Fig. 2. Convergence of the discretisation error of formulation (1), discretisation by the radially symmetry edge shape functions (6).

the discrete counterpart of (2) will redistribute between axial and azimuthal components, albeit underrating the reluctance of the magnetic path. The results indicate that the introduced error is marginal as long as the considered model is truely radially symmetric.

III. Discretisation

An edge shape function $\vec{w}_j(r,\theta,z)$ associated with a radial line $(r_1 \rightarrow r_2, \theta_j, z_j)$ perpendicular to Γ_{ref} is defined by

$$\vec{w_j}(r,\theta,z) = \frac{N_j(r,\theta,z)}{r_2 - r_1}\vec{e_r} \quad ; \tag{6}$$

$$N_j(r,\theta,z) = \frac{a_j + b_j r\theta + c_j \frac{1}{r} z}{2S_e} \quad , \tag{7}$$

where N_j is a scalar function associated with node j and a_j, b_j, c_j and S_e are coefficients such that $N_j(r,\theta_i,z_i) = \delta_{ij}$. The edge shape functions fulfil a *partition-of-unity* property, i.e., their integration along the considered radial line through the associated node j yields 1 [7]. The edge shape functions are *consistent*, i.e., they allow to represent a homogeneous axial field and an azimuthal field with dependence $1/r$ exactly. Formulation (2) is discretized by the Ritz-Galerkin approach, using $\vec{w}_j(r,\theta,z)$ both as test and trial functions.

The consistency and convergence of the discretisation is verified for a model with extent $[r_1,r_2] \times [0,\theta_2] \times [0,z_2]$ and homogeneous material submitted to a set of different excitations (Fig. 2). The consistency of the edge shape functions is reflected by the discretisation error at machine precision for homogeneous axial flux and curl-free azimuthal flux. For the other analytical test cases, the expected convergence of order $\mathcal{O}(h^2) = \mathcal{O}(n^{-1})$ with h the mesh size and n the number of DoFs, is attained.

IV. Application

The 2D FE solver for radially symmetric models is applied to calculate the performance of a three-phase twin-rotor axial-flux PM synchronous machine (Fig. 1) [8]. The no-load electromotive force is calculated by the newly developed 2D radially symmetric FE solver implemented in MATLAB and compared to results from a 3D model constructed and simulated using CST EMStudio [9]. The 2D reduction introduces severe approximations: flux fringing at the inner and outer ends is neglected, as is the case for any 2D model; PM cubes are approximated by bended shapes (thereby preserving the remanent flux); bending magnetic fluxes are treated approximately. Nevertheless, a very good agreement is obtained at a substantially lower computational cost.

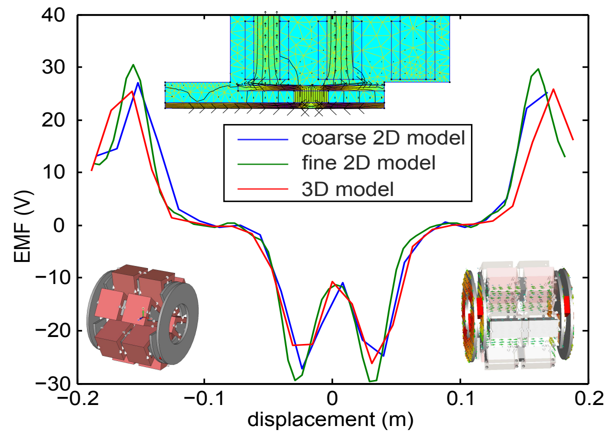

Fig. 3. Electromotive force generated by the disk motor; 3D model and 3D flux pattern (bottom); 2D magnetic flux density (top).

V. Conclusion

Specific edge shape functions are needed for achieving consistency and convergence of a 2D FE discretisation of the magnetic vector potential for radially symmetric models. When applicable, the new solver outperform 3D simulation for a three-phase twin-rotor axial-flux PM synchronous machine.

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