Dual Formulations for Accurate Thin Shell Models in a Finite Element Subproblem Method

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***Abstract*—Subproblem dual finite element magnetostatic and magnetodynamic formulations are developed and compared for correcting the inaccuracies near edges and corners of thin shell models, that replace thin volume regions by surfaces. The surface-to-volume correction problem is defined as one of the multiple subproblems applied to a complete problem, considering successive additions of inductors and magnetic or conducting regions, some of these being thin regions. Each SP requires a proper adapted mesh of its regions, which facilitates meshing and increases computational efficiency.**

I. Introduction

As proposed in [1], [2], thin shell (TS) finite element (FE) models are used to avoid meshing thin regions (TRs) and lighten the mesh of their surrounds. For that, the volume TRs are replaced by surfaces with interface conditions (ICs) linked to 1-D analytical distributions along their thickness that however neglect end and curvature effects. This leads to inaccuracies of field distributions and associated losses near edges and corners, that increase with the thickness. To overcome these disadvantages, a subproblem method (SPM) based on magnetic flux density formulations, proposing a surface-to-volume local correction, has been proposed in [3].

The SPM for TS correction is herein extended to a dual approach for both magnetic field and magnetic flux density formulations, with generalized mesh projections of solutions between the SPs. Also, the SPM naturally allows parameterized analyses of the TR characteristics: permeability, conductivity and thickness. In the proposed SP strategy, a reduced problem (SP u) with only inductors is first solved on a simplified mesh without thin and volume regions. Its solution gives surface sources (SSs) as ICs for added TS regions (SP p), and volume sources (VSs) for possible added volume regions (SP k). The TS solution is then corrected by a volume correction via SSs and VSs that suppress the TS representation and add the volume model.

II. Thin Shell Correction in the Subprobem Method

A. Canonical magnetodynamic or static problem

A canonical magnetodynamic or static problem i, to be solved at step i of the SPM, is defined in a domain Ω_i, with boundary $\partial\Omega_i = \Gamma_i = \Gamma_{h,i} \cup \Gamma_{b,i}$. Material relations and boundary conditions (BCs) are [3]

$$\boldsymbol{h}_i = \mu_i^{-1}\boldsymbol{b}_i + \boldsymbol{h}_{s,i}\,,\ \boldsymbol{j}_i = \sigma_i\boldsymbol{e}_i + \boldsymbol{j}_{s,i} \qquad \text{(1a-b)}$$

$$\boldsymbol{n}\times\boldsymbol{h}_i|_{\Gamma_{h,i}} = \boldsymbol{j}_{f,i}\,,\ \boldsymbol{n}\times\boldsymbol{e}_i|_{\Gamma_{e,i}} = \boldsymbol{k}_{f,i} \qquad \text{(2a-b)}$$

where $\boldsymbol{h}_i$ is the magnetic field, $\boldsymbol{b}_i$ is the magnetic flux density, $\boldsymbol{e}_i$ is the electric field, $\boldsymbol{j}_i$ is the electric current density, μ_i is the magnetic permeability, σ_i is the electric conductivity and $\boldsymbol{n}$ is the unit normal exterior to Ω_i. The notation $[\cdot]_{\gamma_i} = \cdot|_{\gamma_i^+} - \cdot|_{\gamma_i^-}$ refers to the discontinuity of a quantity through an interface γ_i (with sides γ_i^+ and γ_i^-) in Ω_i, defining ICs. The fields $\boldsymbol{h}_{s,i}$ and $\boldsymbol{j}_{s,i}$ in (1 a) and (1 b) are VSs that can be used for expressing changes of a material property in a volume region, from μ_p and σ_p for SP p to μ_k and σ_k for SP k [3], i.e.

$$\boldsymbol{h}_{s,k} = (\mu_k^{-1} - \mu_p^{-1})\boldsymbol{b}_p\,,\ \boldsymbol{j}_{s,k} = (\sigma_k - \sigma_p)\boldsymbol{e}_p \qquad (3)$$

with $\mu_p = \mu_0$, $\mu_k = \mu_{volume}$, $\sigma_p = 0$ and $\sigma_k = \sigma_{volume}$. The fields $\boldsymbol{j}_{f,i}$ and $\boldsymbol{k}_{f,i}$ in (2 a) and (2 b) are SSs. They define possible SSs that account for particular phenomena occuring in the idealized TR between γ_i^+ and γ_i^- [3]. This is the case when some field traces in SP p are forced to be discontinuous (e.g. in TS model), whereas their continuity must be recovered via an SP k, which is done via a SS in SP k fixing the opposite of the trace discontinuity solution of SP p.

B. SSs via ICs for subproblems

The solution of an SP u is first known for a particular configuration, e.g. for an inductor alone (Fig. 1, a), or more generally resulting from the superposition of several SP solutions. The next SP p consists in adding a TS to this configuration (Fig. 1, b). From SP u to SP p, the solution u gives SSs for the added TS γ_p, through TS ICs [2]. The

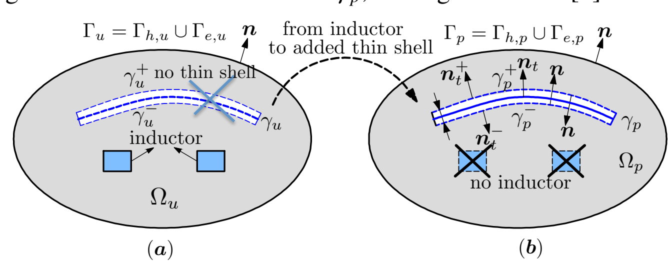

Figure 1: Interface condition between SP u and SP p.

$\boldsymbol{b}$-formulation uses a magnetic vector potential $\boldsymbol{a}_i$ (such that curl $\boldsymbol{a}_i = \boldsymbol{b}_i$), split as $\boldsymbol{a} = \boldsymbol{a}_{c,i} + \boldsymbol{a}_{d,i}$ [2]. The $\boldsymbol{h}$-formulation uses a similar splitting for the magnetic field, $\boldsymbol{h}_i = \boldsymbol{h}_{c,i} + \boldsymbol{h}_{d,i}$. The fields $\boldsymbol{a}_{c,i}$, $\boldsymbol{h}_{c,i}$ and $\boldsymbol{a}_{d,i}$, $\boldsymbol{h}_{d,i}$ are continuous and discontinous respectively through the TS. The trace discontinuities in SP p

$[\boldsymbol{n}\times\boldsymbol{h}_p]_{\gamma_p}$ and $[\boldsymbol{n}\times\boldsymbol{e}_p]_{\gamma_p}$ (with $\boldsymbol{n}_t = -\boldsymbol{n}$) can be expressed as

$$[\boldsymbol{n}\times\boldsymbol{h}_p]_{\gamma_p} = [\boldsymbol{n}\times(\boldsymbol{h}_u+\boldsymbol{h}_p)]_{\gamma_p} - [\boldsymbol{n}\times\boldsymbol{h}_u]_{\gamma_p} \quad = [\boldsymbol{n}\times\boldsymbol{h}]_{\gamma_p} \quad (4)$$

$$[\boldsymbol{n}\times\boldsymbol{e}_p]_{\gamma_p} = [\boldsymbol{n}\times(\boldsymbol{e}_u+\boldsymbol{e}_p)]_{\gamma_p} - [\boldsymbol{n}\times\boldsymbol{e}_u]_{\gamma_p} \quad = [\boldsymbol{n}\times\boldsymbol{e}]_{\gamma_p} \quad (5)$$

because there are no discontinuities in SP u (before adding γ_p). In addition, one has TS-ICs in both formulations [2]

$$[\boldsymbol{n}\times(\boldsymbol{h}_u+\boldsymbol{h}_p)]_{\gamma_p} = [\boldsymbol{n}\times\boldsymbol{h}_p]_{\gamma_p} = \mu_p\beta_p\,\partial_t(2\boldsymbol{a}_{c,p}+\boldsymbol{a}_{d,p}) \quad (6)$$

$$[\boldsymbol{n}\times(\boldsymbol{e}_u+\boldsymbol{e}_p)]_{\gamma_p} = [\boldsymbol{n}\times\boldsymbol{e}_p]_{\gamma_p} = \mu_p\beta_p\,\partial_t(2\boldsymbol{h}_{c,p}+\boldsymbol{h}_{d,p}) \quad (7)$$

with β_p given in [2]. The resulting FE formulations are then written for SPs u, p and k, which will be developed in the full paper.

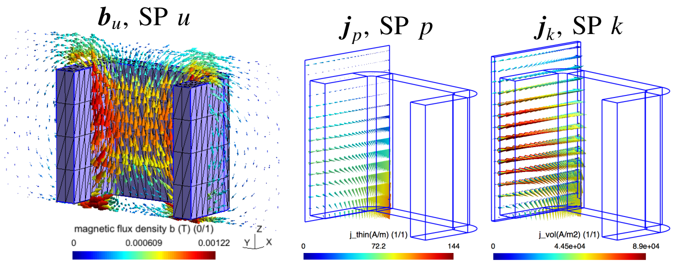

Figure 2: TEAM problem 21 (1/4th of the geometry): magnetic flux density $\boldsymbol{b}_u$ (in a cut plane) generated by a stranded inductor (*left*), eddy current density $\boldsymbol{j}_p$ on TS model (*middle*) and its volume correction $\boldsymbol{j}_k$ (*right*) (thickness d = 10 mm).

III. Application Example

A 3-D test problem is based on TEAM problem 21 (model B, coil and plate, Fig. 2). An SP scheme considering three procedures is developed. A first FE SP u with the stranded inductors alone is solved on a simplified mesh without any TR (Fig. 2, *left*). Then an SP p is solved with the added TR via a TS FE model (Fig. 2, *middle*). At last, an SP k replaces the TS FEs with the actual volume FEs (Fig. 2, *right*). The TS error on $\boldsymbol{j}_p$ locally reaches 83% (Fig. 2, *middle*), with f = 50 Hz, μ = 200 and σ = 6.484 MS/m (skin depth δ = 6.15 mm). The inaccuracies on the Joule power loss densities of TS SP p are pointed out by the importance of the correction SP k (Fig. 3, *top*). Significant errors on TS SP p along the horizontal half inner width (y-direction) reach 85% near the plate ends (Fig. 3, *top*), with δ = 2.1 mm and thickness d = 7.5 mm. For d = 1.5 mm, they are reduced to below 10%. In particular, accurate local corrections with volume correction SP k are checked to be close to the solution of the complete problem, with errors lower than 0.1% (Fig. 3, *bottom*).

Table I shows the Joule losses in the plate with an approximate BC for SP k. The exterior boundary of SP k is first chosen at a distance D_{bound} = 200d from the TR, with thickness d = 10 mm. The inaccuracies on Joule losses for TS SP p reach 76.4%, or 1.2% for volume correction SP k, with f = 50 Hz, μ = 100 and σ = 6.484 MS/m in both cases. The proposed SP strategy allows to locally focus on the mesh of volume correction SP k and its neighborhood. It is shown that even if D_{bound} is reduced to 2d, the error on SP k is 1.53%, which is still very accurate. For d = 2 mm, the errors on Joule losses for SP p are reduced to 6.09%, or 0.05% for SP k.

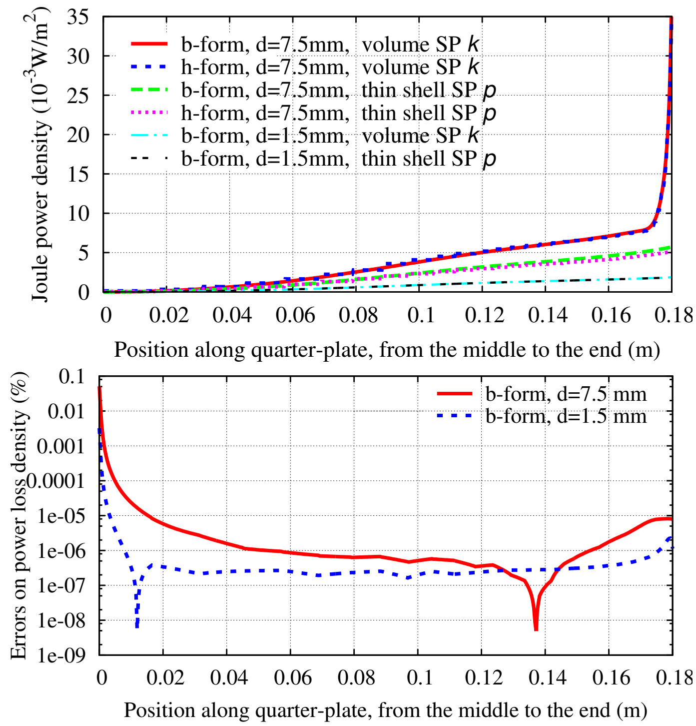

Figure 3: Power loss density with TS and volume solutions (*top*); errors on the power loss density between volume correction SP k and complete problem (*bottom*), along horizontal half inner width (y-direction), with effects of d (f = 50 Hz, μ_r = 200, σ = 6.484 MS/m).

The SPM allows to correct the inaccuracies proper to the TS model. Accurate eddy current and power loss densities are obtained, especially along the edges and corners of the TRs, also for significant thicknesses. The refined mesh for volume correction can be reduced to a close neighborhood of the TR.

Table I: Joule losses in the plate with approximate BCs (f = 50 Hz, μ = 100, σ = 6.484 MS/m), with $\boldsymbol{b}$−formulation.

D_{bound}	d = 10 mm (thickness of the plate)			Errors %	
	Thin shell P_{thin} (W)	Volume P_{vol} (W)	Reference P_{ref} (W)	Between P_{thin} and P_{ref}	Bet P_{vol} and P_{ref}
200d	0.0114	0.0477	0.0483	76.4	1.2
20d	0.0114	0.0476	0.0483	76.4	1.35
2d	0.0114	0.0475	0.0483	76.4	1.53
	d = 1 mm (thickness of the plate)				
200d	0.0108	0.0115	0.0115	6.09	0.00
2d	0.0108	0.011506	0.0115	6.09	0.05

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