Calculation of 3D magnetic fields produced by MHD active control systems in fusion devices

Paolo Bettini[1,2], Ruben Specogna[3]
[1]Università di Padova, Dipartimento di Ingegneria Industriale (DII), Via Gradenigo 6/A, 35131 Padova, Italy
[2]Consorzio RFX, EURATOM-ENEA Association, C.so Stati Uniti 4, 35127 Padova, Italy
[3]Università di Udine, Dipartimento di Ingegneria Elettrica, Gestionale e Meccanica, Via delle Scienze 206, 33100 Udine, Italy
paolo.bettini@unipd.it

***Abstract*—In a magnetic confinement fusion device the presence of a conductive shell surrounding the plasma is important to guarantee a good MHD stability; however, a feedback control system of the plasma instabilities is also required. Recently, a very effective control scheme, named *Clean Mode Control* (CMC), has been proposed in RFX-mod. The CMC is based on the real-time correction (cleaning), under ideal shell hypothesis, of the sideband harmonics in the magnetic field produced by the discrete local active coils. In this paper we describe how to exploit periodic boundary conditions in the frequency domain calculations of 3D magnetic fields produced by MHD active control systems, in view of a possible development of the CMC algorithm for more realistic shell geometry.**

***Index Terms*—magnetohydrodynamic, sidebands, clean mode control, periodic boundary conditions**

I. Introduction

In a magnetic confinement fusion device the presence of a conductive wall (shell) surrounding the plasma is important to guarantee a good magnetohydrodynamic (MHD) stability. However, due to the finite diffusion time of any resistive shell, the tearing modes (TM) radial field penetrates the shell, its amplitude considerably increases and causes severe plasma-wall interactions which can lead to premature termination of the discharges [1]. To overcome this problem a feedback system is also required to control the edge radial field by means of a set of local active coils. On the other hand, due to the discrete nature of the local coils, an infinite sequence of sideband harmonics in the magnetic field is also produced. Therefore, if the magnetic sensors and the coils share the same periodicity, as in the *Intelligent Shell* (IS) scheme [2], the aliasing of the sidebands determines a systematic error on the Fourier spectrum of the measurements used as the feedback variables. Recently, a very effective control scheme, named *Clean Mode Control* (CMC) [3], [4], has been proposed in RFX-mod, a toroidal device (major radius $R_0 = 2m$, minor radius $a = 0.459m$) which is equipped with a sophisticated MHD feedback control system composed of $48x4$ independently driven active coils that completely cover the torus. The CMC scheme is based on the real-time correction (cleaning) of the measurements from the high periodicity sidebands produced by the active coils: the feedback variables are not the raw measurements, as in the IS, but the poloidal and toroidal m, n Fourier harmonics related to TMs, estimated as much correctly as possible with the sidebands subtraction, by means of standard formulas for the radial magnetic field in cylindrical geometry, expressed in terms of the modified Bessel Functions I_p, K_p, according to the thin-shell dispersion relation [3].

In this paper, after a brief description of the CMC scheme, we focus on the exploitation of periodic boundary conditions for the calculation in the frequency domain of 3-D magnetic fields produced by MHD active control systems in fusion machines. Then we present some preliminary results in view of possible optimization of the CMC scheme, on the basis of a more detailed model of the conducting structures surrounding the plasma.

II. Clean Mode Control

To recall the idea behind the CMC scheme, it is convenient to refer to the single shell configuration in cylindrical geometry (large aspect ratio approximation), which allows to investigate all the basic aspects of the problem.

A cylindrical coordinate system $(r, \theta, \phi \equiv z/R_0)$ is adopted. The plasma (minor radius $r = a$), is contained within a uniform resistive shell (minor radius $r = b$, thickness δ). The model includes a grid of $N \times M$ active coils outside the shell ($r = c$), and a grid of $N \times M$ radial field sensors inside the shell ($r = b$). Both the coils and the sensors are saddles fully covering the torus, i.e. rectangles of poloidal (azimuthal) extent $\Delta\theta = 2\pi/M$ and toroidal (longitudinal) extent $\Delta\phi = 2\pi/N$, centered at the angles $\theta_i = (i-1)\,\Delta\theta$, $i = 1, \ldots M$, $\phi_j = (j-1)\,\Delta\phi$, $j = 1, \ldots N$.

In this framework, a set of discrete Fourier transform (DFT) harmonics of fields $b^{m,n}_{r,DFT}$ can be univocally defined in terms of radial field measurements $b^r_{i,j}$ as

$$b^{m,n}_{r,DFT} = \frac{1}{M\,N} \sum_{\substack{i=1,M\\ j=1,N}} b^r_{i,j}\, e^{-i\,(m\theta_i + n\phi_j)}. \tag{1}$$

On the other hand, due to their discrete nature, the active coils produce an infinite sequence of sidebands in the radial field, which enter in the DFT harmonics as

$$b^{m,n}_{r,DFT} = \sum_{\substack{p=m+l\,M\\ q=n+kN\\ \{l,k\}\in Z}} b^{p,q}_r\, f(p,q), \tag{2}$$

where the shape factor $f(p,q)$ is

$$f(p,q) = \sin\left(q\frac{\Delta\phi}{2}\right) / \left(q\frac{\Delta\phi}{2}\right) \sin\left(p\frac{\Delta\theta}{2}\right) / \left(p\frac{\Delta\theta}{2}\right)$$

The CMC scheme is based on the real-time de-aliasing (cleaning) of the measurements from the high periodicity sidebands: the feedback variables are not the raw measurements, as in the IS, but the poloidal and toroidal m,n Fourier harmonics related to TMs, estimated as much correctly as possible with the numerical subtraction of the sidebands in (2). The terms $b_r^{p,q}$ can be calculated using the standard vacuum solution for the radial magnetic field in cylindrical geometry, by means of the modified Bessel Functions I_p, K_p, according to the thin-shell dispersion relation [3].

III. Discrete Geometric Formulation

A discrete geometric formulation for eddy-current problems in the frequency domain is presented, which is based on the circulation of the magnetic vector potential and exploits periodic boundary conditions over hexahedral grids.

The 3-D domain of interest $\mathcal{D}$ is covered by a mesh of generic hexahedra, whose incidences are encoded in the *cell complex* $\mathcal{K}$ represented by the standard incidence matrices **G**, **C** and **D** [5]. A dual barycentric complex $\tilde{\mathcal{K}}$ is obtained from $\mathcal{K}$ by using the *barycentric subdivision*; its incidence matrices are $\tilde{\mathbf{G}} = \mathbf{D}^T$, $\tilde{\mathbf{C}} = \mathbf{C}^T$ and $\tilde{\mathbf{D}} = -\mathbf{G}^T$.

Three subdomains of $\mathcal{D}$ are identified: the passive conductive region $\mathcal{D}_c$ (e.g the shell and/or other conducting structures surrounding the plasma), the nonconductive region $\mathcal{D}_a$ (air or vacuum), and the source region $\mathcal{D}_s$ (e.g. the active coils used to control the plasma instabilities).

By combining the discrete Ampère's law and Faraday's law with the discrete counterpart of the constitutive laws for the flux density **B** and the current density **J**, a symmetric complex linear system of equations is obtained [6],

$$\begin{aligned} &(\mathbf{C}^T \boldsymbol{\nu}\, \mathbf{C})\, \mathbf{A}_r = \mathbf{0}, && \forall e \in \mathcal{D}_a \bigcup \mathcal{D}_s \\ &(\mathbf{C}^T \boldsymbol{\nu}\, \mathbf{C} + i\omega\, \boldsymbol{\sigma})\, \mathbf{A}_r = -i\omega\, \boldsymbol{\sigma}\, \mathbf{A}_s, && \forall e \in \mathcal{D}_c \end{aligned} \tag{3}$$

where ω is the angular frequency, $\boldsymbol{\nu}$ and $\boldsymbol{\sigma}$ are square matrices that require metric notions, material properties, and some hypothesis on the fields in order to be computed. The unknowns $\mathbf{A}_r$ are the circulations of the magnetic vector potential along primal edges $e \in \mathcal{D}$ due to eddy currents in $\mathcal{D}_c$, only. On the RHS, $\mathbf{A}_s$ denotes the circulations of the magnetic vector potential along $e \in \mathcal{D}_c$ produced by the sources in $\mathcal{D}_s$, only; each entry of $\mathbf{A}_s$ can be computed with standard closed formulas. Then, the circulations of the magnetic vector potential **A** can be expressed as: $\mathbf{A} = \mathbf{A}_r + \mathbf{A}_s$.

A. Periodic boundary conditions

We assume a grid of $N \times M$ coils to be periodically distributed in space surrounding a resistive shell with the same geometry described in section II. The numerical domain $\mathcal{D}$ is delimited by a cylindrical surface (minor radius $r = d \gg c$) and two boundary planes ($z = 0$, $z = 2\pi R_0$). The hexahedra

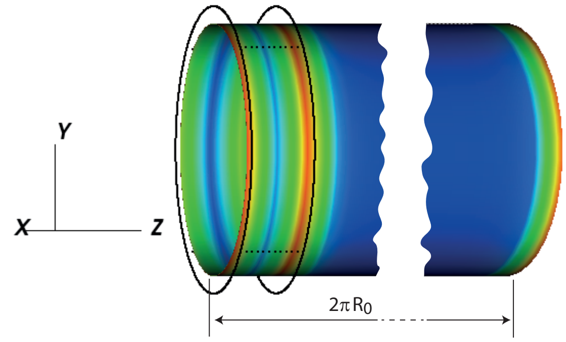

Figure 1: The module of the current density induced on a uniform resistive shell is shown. In this example, out of 48×4, only the first 4 saddle coils are active.

mesh is constructed by the sweeping of a 2D fine quadrilateral mesh.

The flux density distribution on the two planes are identical due to the symmetry of the conducting structures and the periodicity of the sources (it is intrinsic in toroidal devices and must be imposed in the cylindrical geometry used to derive the formulation of the CMC scheme). Then, the same DoFs $\mathbf{A}_r$ are imposed on corresponding edges of the periodic boundary planes; the Boundary Conditions (BCs) are completed by imposing $\mathbf{A}_r = 0$ on each primary edge of the cylindrical surface.

IV. Numerical results

The proposed approach has been applied for the calculation, of the 3-D magnetic fields (virtual measurements) produced by a grid of 48×4 coils in the presence of either one single uniform shell (with the geometry illustrated in section II) or a more realistic configuration (e.g. including two coaxial conducting structures and taking into account the effects of the gaps for which analytical solutions are not available).

In Fig. 1, the module of the current density induced on a uniform resistive shell is shown; in the example, out of 48×4, only the first 4 saddle coils are active (fed with equal sinusoidal currents at $100\,Hz$). The correctness of the implementation is confirmed by the eddy currents on the right side, being the only sources placed on the opposite side of the domain. In the full paper more results in the frequency domain (from $10Hz$ to $100Hz$) will be presented and discussed.

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