Hierarchical Block Wavelet Compression of 3-D Eddy Current Problems

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***Abstract*—In this paper the hierarchical block wavelet compression is modified and extended for the compression of the fully populated system matrix of eddy current problems. As the system is bad conditioned, an iterative regularization method is implemented, which enables high compression rates and reduced compression errors for large problems. A comparative study between the Block Wavelet Compression and the Hierarchical Block Wavelet Compression as well as the speed-up of iterative regularization methods compared to the Tikhonov regularization will be shown for 3D eddy current problems using an integral formulation.**

***Index Terms*—Block wavelet compression, eddy currents, volume integral equation method, regularization, hierarchical matrices.**

I. Introduction

The calculation of large eddy current problems using integral equations requires a compression technique due to the fully populated system matrix. In previous works the Block Wavelet Compression (BWC) was successfully implemented for radiative heat transfer problems [1] and eddy current problems [2]. This reduces the memory cost from quadratic to almost linear dependency for a high number of degrees of freedom (DoFs). A big advantage of this technique is the application to system matrices with an arbitrary number of DoFs, which is not the case for conventional wavelet based compression techniques. But the disadvantage is that the geometry of the model is not taken into account.

Therefore, the Hierarchical Wavelet Compression (HWC) [3] is modified for the calculation of eddy current problems. Taking the geometry model into account leads to smaller compression errors for equal compression rates.

Due to the high condition number of the system matrix, an iterative regularization step [5] is implemented. This step is mandatory as even small compression rates lead to an unsolvable system of linear equations.

II. Eddy Current Formulation

Eddy currents in non-magnetic, electrically conductive region V_c can be described by the following equation [4]:

$$\frac{1}{\kappa}\boldsymbol{J}(\boldsymbol{r})+\frac{\partial}{\partial t}\frac{\mu_0}{4\pi}\int_{V_c}\frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}V'=-\frac{\partial}{\partial t}\boldsymbol{A}_s(\boldsymbol{r}). \qquad (1)$$

$\boldsymbol{J}$ is the eddy current density, κ the electrical conductivity, μ_0 the magnetic field constant, and $\boldsymbol{A}_s$ the magnetic vector potential of a time-varying source.

Introducing edge-element-based shape functions $\boldsymbol{N}_i$ and their curls $\nabla\times\boldsymbol{N}_i$ and applying the Galerkin method, the eddy current problem is described by the linear system [4],[6]

$$\{U\}=[R]\{T\}+[L]\frac{\mathrm{d}\{T\}}{\mathrm{d}t}, \qquad (2)$$

with

$$R_{ij}=\int_{V_c}\frac{1}{\kappa}\nabla\times\boldsymbol{N}_i(\boldsymbol{r})\cdot\nabla\times\boldsymbol{N}_j(\boldsymbol{r})\,\mathrm{d}V, \qquad (3)$$

$$L_{ij}=\frac{\mu_0}{4\pi}\int_{V_c}\int_{V_c'}\frac{\nabla\times\boldsymbol{N}_i(\boldsymbol{r}')\cdot\nabla\times\boldsymbol{N}_j(\boldsymbol{r})}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}V'\mathrm{d}V, \qquad (4)$$

$$U_i=-\frac{\partial}{\partial t}\int_{V_c}\boldsymbol{A}_s\cdot\nabla\times\boldsymbol{N}_i(\boldsymbol{r})\,\mathrm{d}V. \qquad (5)$$

The obtained solution of the system of linear equations is the integral of the electric vector potential $\boldsymbol{T}$ over the element edges of the discretized eddy current region. The eddy current density inside an element is obtained by derivation

$$\boldsymbol{J}(\boldsymbol{r})=\sum_{i=1}^{n}\int_{V_c}\nabla\times\boldsymbol{N}_i(\boldsymbol{r})T_i\mathrm{d}V. \qquad (6)$$

V and V' are the volumes of the elements belonging to the evaluation edge and the source edge, $\nabla\times\boldsymbol{N}_i(\boldsymbol{r})$ is the curl of the edge shape function of edge i, and T_i the solution value of this edge.

III. Regularization

Due to the high condition number of the system matrix, even small compression rates lead to very high compression errors or unsolvable systems. Therefore, the use of a regularization technique after compression is mandatory. This gives low errors at high compression rates even for small-sized problems. In [2] the Block Wavelet Compression coupled with the Tikhonov regularization was applied to eddy current problems. The regularization parameter is calculated selectively by the L-curve method [5] or U-curve method [7]. The disadvantage of the Tikhonov regularization is the matrix-matrix-multiplication, which is not practical for large problems. Therefore, iterative regularization methods [5], which solves the system at the same time, are implemented in our software ELFE++. The programming language of this software is C++.

IV. Compression

As the HWC is based on the BWC, a short introduction to the BWC will be given before the modified and improved HWC will be explained.

A. Block Wavelet Compression

At the beginning, the system matrix is split into k blocks of size $2^p \times 2^q$, where $p, q \in \mathbb{N}$. The size of the system matrix corresponds to a reduced number of edges, using the tree-cotree decomposition [6]. After the splitting, each block is transformed by a 2-D FWT and thresholded in wavelet space $V_{p,q}$ to thin out the coefficients [1]. To solve the eddy current problem over the various number of k blocks at compressed state, the right hand side (RHS) and the solution vector are also split accordingly to the horizontal or vertical block structure and afterwards transformed by a 1-D FWT [1]. The eddy current problem is solved by using an iterative solver like GMRES or CG for all compressed sub spaces $V_{p,q}$.

B. Modified Hierarchical Block Wavelet Compression

The HWC extends the BWC by taking the geometry into account. Hereby, the geometrical model is fragmented by *kdtree*, which groups the edges and allows for fast calculation of distances between the groups. Hence, the indices i, j covered by a block are mapped back into geometry and by the kd-tree shown how large the distance between source and destination is. Small distances indicate important integral results due to a decay of the integral kernel of $1/dist$ and the block is refined by hierarchical matrices [3]. The gain of the HWC is to predict the importance of areas in the system matrix, which allows for an adjustable compression rate. The iterative solving is performed in hierarchical wavelet sub spaces in common to an extended scheme of the BWC.

To get a smooth decay of the system matrix, the DoFs are sorted at the beginning using a three-dimensional binary *kd-tree* [1]. The sorting plays a key role regarding the compression and has contrary effects for the BWC and HWC. More details about this will be given in the full paper.

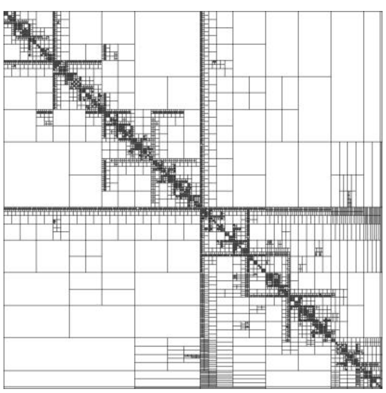

Fig. 1. Hierarchical matrix of the eddy current problem shown in Figure 2

In Figure 1 the hierarchical matrix of the eddy current problem of Figure 2 is shown. One can see that along the diagonal the size of the block is small, meaning that these blocks contain important values and must not be compressed. The matrix structure is obtained by sorting the DoFs.

The final step in this compression techniques is to reverse transform the solution of the problem into the normal space. BWC and HWC reduce the memory costs from quadratic to linear dependency. The computational cost of the transform can be neglected.

V. Preliminary Results

The efficiency of the HWC will be demonstrated on a example of a copper plate and a coil above it (Fig. 2).

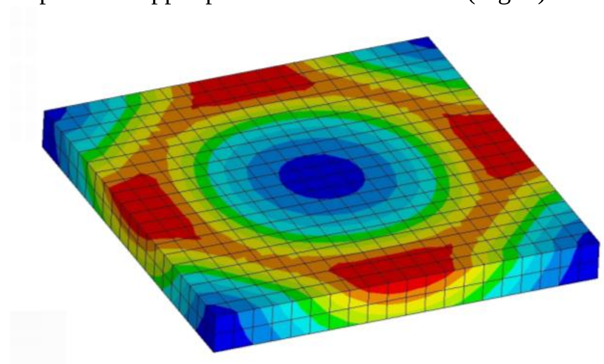

Fig. 2. Numerical result of the compressed case by the HWC

For better visualization of the result, only the plate is shown. The plate has a length of 0.2 m and a height of 0.02 m. It is meshed by 800 hex8-elements, which results in 3402 edges, 1323 nodes and 1481 DoFs (edges).

Fig. 2 shows the eddy current density on the plate after $10 \cdot 10^{-3}$ seconds for a compression rate of 80 % using the HWC. The eddy current is caused by a changing coil current of 500 A/s. The relative error between the compressed and uncompressed case is 1.8 %. This is a very good result regarding the number of DoFs and the compression rate. As the error is hardly visible, the solution of the uncompressed case is not shown.

Further results and comparisons between the BWC and HWC, as well as the efficiency of the iterative regularization methods will be shown for larger problems in the full paper.

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