Nonlinear Computational Homogenization Method for the Evaluation of Eddy Currents in Soft Magnetic Composites

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Abstract—**In this paper, a heterogeneous multiscale method (HMM) based technique is applied to model the behaviour of electromagnetic fields in soft magnetic composites (SMC). Two problems are derived from the two-scale homogenization theory: a macroscale problem that captures the slow variations of the overall solution, and many mesoscale problems that allow determining the constitutive laws at the macroscale. As application, an SMC core is considered.**

Index Terms—**Composite materials, multiscale homogenization, finite element methods, eddy currents, magnetic hysteresis.**

I. Introduction

The use of the soft magnetic composites (SMC) in electric devices has recently increased. These materials, made from a metallic powder compacted with a dielectric binder, are a good alternative to laminated ferromagnetic structures as their granular mesoscale structure allows to significantly reduce the eddy current losses. Furthermore unlike the laminated ferromagnetic structures, SMC exhibit isotropic magnetic properties what makes them good candidates for manufacturing machines with 3-D flux paths.

The use of classical numerical methods such as the finite element (FE) method to study the behaviour of SMC is computationally very expensive. Indeed a very fine mesh is required to capture fine scale variations, i.e. variations at the level of metallic grains, whence the need of multiscale methods. The application of multiscale methods to study SMC is quite recent. In these methods, a cell problem is solved on an elementary cell and the solution is used to compute the homogenized constitutive laws (electric and magnetic) [1], [2], [3]. The choice of the elementary cell is also crucial in order to accurately model real SMC structures.

In this paper, we extend the computational homogenization method successfully used for modeling laminated ferromagnetic cores [4] to the case of SMC. The method is based on the heterogeneous multiscale method (HMM) [5] and couples two types of problems: a macroscale problem that captures the slow variations of the overall solution, and many mesoscale problems that allow to determine the constitutive laws at the macroscale.

II. Multiscale computational homogenization model

Let the superscript $\varepsilon = l/L$ be the ratio between the finest scale l and the scale of the material or the characteristic length of external loadings L, and denote quantities with rapid spatial variations. The vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ denote the macroscale and the mesoscale spatial positions. We also define differential operators with respect to these positions, e.g. curl_x and curl_y are curl operators with respect to $\boldsymbol{x}$ and $\boldsymbol{y}$. The subscripts M, m and c refer to the macroscale, the total and the correction mesoscale quantities, respectively.

Further, we exploit the two-scale convergence theory [6] to develop the homogenized model for the Maxwell equations in magnetodynamics. This model is derived as the limit of electromagnetic fields and operators in the Maxwell equations for $\varepsilon \to 0$, which holds if the nonlinear magnetic mapping is maximal monotone [7] or can be derived from the minimization of a lower semi-continuous convex functional [6]. In practice, the homogenized model has already been used for solving hysteretic problems [4].

We replace the fine-scale problem with rapidly fluctuating fields by a macroscale problem defined on a coarse mesh covering the entire domain and many mesoscale problems defined each on a small, finely meshed area around the Gauss macroscale points [4]. We use the $\boldsymbol{a} - v$ magnetodynamic formulation for both scales.

The mesoscale problems are governed by the following weak forms: find $\boldsymbol{a}_c^\varepsilon$ and v_c^ε such that

$$\left(\boldsymbol{\mathcal{H}}^\varepsilon(\text{curl}_y\, \boldsymbol{a}_c^\varepsilon + \boldsymbol{b}_M), \text{curl}_y\, \boldsymbol{a}_c^{'\varepsilon}\right)_{\Omega_m} + \left(\sigma_m^\varepsilon \partial_t \boldsymbol{a}_c^\varepsilon, \boldsymbol{a}_c^{'\varepsilon}\right)_{\Omega_{m_c}} + \left(\sigma_m^\varepsilon \text{grad}_y\, v_c^\varepsilon, \boldsymbol{a}_c^{'\varepsilon}\right)_{\Omega_{m_c}} = \left(\sigma_m^\varepsilon(\boldsymbol{e}_M - \kappa \partial_t \boldsymbol{b}_M \times \boldsymbol{y}), \boldsymbol{a}_c^{'\varepsilon}\right)_{\Omega_{m_c}} \quad (1)$$

$$\left(\sigma_m^\varepsilon \partial_t \boldsymbol{a}_c^\varepsilon, \text{grad}_y\, v_c^{'\varepsilon}\right)_{\Omega_{m_c}} + \left(\sigma_m^\varepsilon \text{grad}_y\, v_c^\varepsilon, \text{grad}_y\, v_c^{'\varepsilon}\right)_{\Omega_{m_c}} = \left(\sigma_m^\varepsilon(\boldsymbol{e}_M - \kappa \partial_t \boldsymbol{b}_M \times \boldsymbol{y}), \text{grad}_y\, v_c^{'\varepsilon}\right)_{\Omega_{m_c}} + \left\langle \boldsymbol{n} \cdot \boldsymbol{j}_M, v_m^{'\varepsilon}\right\rangle_{\Gamma_{m_c}} \quad (2)$$

hold for all test functions $\boldsymbol{a}_c^{'\varepsilon}$ and $v_c^{'\varepsilon}$ in appropriate function spaces. Domains Ω_m, Ω_{m_c} and Γ_{m_c} are the entire mesoscale domain, the conducting part of the mesoscale domain and the boundary of Ω_{mc}. The mesoscale magnetic field is given by $\boldsymbol{h}_m^\varepsilon = \boldsymbol{\mathcal{H}}^\varepsilon(\boldsymbol{b}_m^\varepsilon)$ and σ_m^ε is the electric conductivity tensor. The macroscale fields $\boldsymbol{b}_M$, $\boldsymbol{e}_M$ and $\boldsymbol{j}_M$, namely, the magnetic flux density, the electric field and the current density are imposed source terms determined by the macroscale problem and *downscaled* (exchange of information from macro to mesoscale). The constant κ equals $1, 1/2$ for 2-D and 3-D problems, respectively. Periodic boundary conditions must also be imposed for the electric scalar potential and the tangential component of the magnetic vector potential [4].

The macroscale problem is defined by the following weak forms: find $\boldsymbol{a}_M$ and v_M such that

$$(\boldsymbol{h}_M, \mathrm{curl}_x\, \boldsymbol{a}'_M)_{\Omega} + (\sigma_M \partial_t \boldsymbol{a}_M, \boldsymbol{a}'_M)_{\Omega_c} + \left(\sigma_M \mathrm{grad}_x\, v_M, \underline{a}'_M\right)_{\Omega_c} = \left(\boldsymbol{j}_s, \underline{a}'_M\right)_{\Omega_s} \quad (3)$$

$$(\sigma_M \partial_t \boldsymbol{a}_M, \mathrm{grad}_x\, v'_M)_{\Omega_c} + (\sigma_M \mathrm{grad}_x\, v_M, \mathrm{grad}_x\, v'_M)_{\Omega_c} = 0 \quad (4)$$

hold for all test functions $\boldsymbol{a}'_M$ and v'_M in appropriate function spaces. Domains Ω, Ω_c and Ω_s denote the entire domain, the conducting domain and the domain of inductors, respectively. The macroscale terms $\boldsymbol{h}_M = \mathcal{H}_M(\boldsymbol{b}_M)$, $\boldsymbol{j}_s$ and σ_M are the magnetic field, the source current density and the electric conductivity tensor, respectively. The *upscaling* (exchange of information from mesoscale to macroscale) consists then in computing the missing constitutive laws σ_M, $\boldsymbol{h}_M$ together with the tangent operator $\partial \mathcal{H}_M / \partial \boldsymbol{b}_M$ at the macroscale using the mesoscale fields. We apply the asymptotic expansion theory to homogenize once and for all the linear electric conductivity [8]. To upscale the nonlinear magnetic law, we average the nonlinear magnetic field obtained by solving cell problems around the Gauss points of the macroscale mesh.

After time discretization of equations (1)–(4) the solution is obtained iteratively using the Newton–Raphson method at each level. For a given time step, we solve a macroscale problem and as many mesoscale problems as Gauss points we have in the macroscale mesh. The macroscale and mesoscale problems exchange information (*downscaling* and *upscaling*) iteratively till the convergence of the macroscale problem.

III. Application

An SMC material has been modeled using a 2-D geometry with 10×10 square elementary cells of $5.5 \times 5.5\,\mathrm{mm}^2$ each. Each cell comprises a conducting material surrounded by an insulating layer that represents the dielectric binder (Fig. 1 – top). Only half of the geometry is used thanks to the symmetry of the problem.

The insulation material is linear isotropic (with $\mu_r = 1$ and $\sigma = 0$). The conductor has an isotropic electric conductivity $\sigma = 5\,\mathrm{MS}$ and is governed by the nonlinear magnetic law $\mathcal{H}^{\varepsilon}(\boldsymbol{b}^{\varepsilon}) = \left(\alpha + \beta\, \exp(\gamma \|\boldsymbol{b}^{\varepsilon}\|^2)\right) \boldsymbol{b}^{\varepsilon}$ with $\alpha = 388$, $\beta = 0.3774$ and $\gamma = 2.97$. A sinusoidal electric current density with frequency $f = 5\,\mathrm{kHz}$ and amplitude $\boldsymbol{j}_s = 5\,10^8\,\mathrm{A/m}^2$ is imposed in inductors located at the left, the right and the top of the SMC material (see Fig. 1 – top). The reference solution is obtained by solving a FE problem on an extremely fine mesh of the whole SMC structure with 178 118 elements. The mesoscale problems are solved on a square elementary cell meshed with 3 236 elements (Fig. 1 – top).

Results in (Fig. 1 – bottom) show a good agreement between the reference solution (labelled "Ref") and the local mesoscale solutions (labelled "Meso_1, Meso_2 and Meso_3").

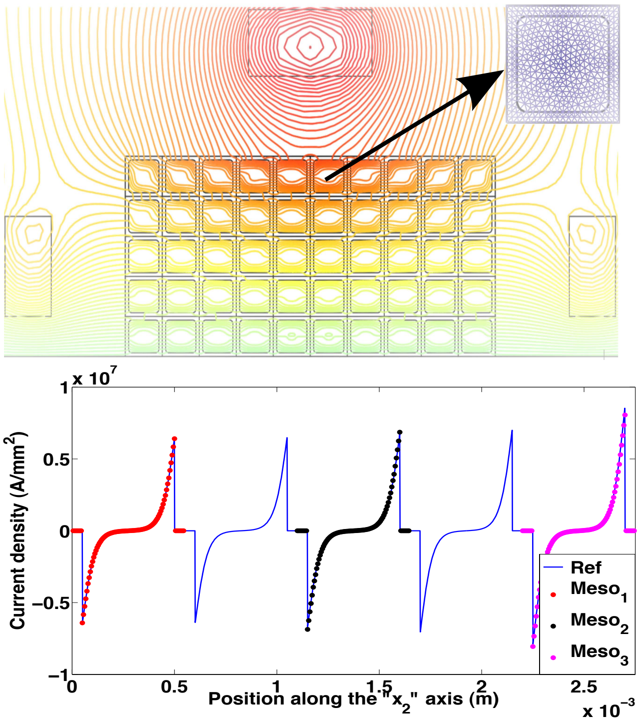

Figure 1. Top: half of the geometry and magnetic vector potential $\underline{a}$ (x_3 component). Mesh of the square elementary cell, meso domain, on the right upper corner. Bottom: Eddy currents $\boldsymbol{j}^{\varepsilon}_m$ (x_1 component) for a cut at $x_1 = 0.0825\,\mathrm{mm}$ ($t = 0.00002\mathrm{s}$). Comparison between the FE reference model and 3 mesoscale solutions defined with x_2 in intervals $[0, 0.055]\,\mathrm{mm}$, $[0.11, 0.165]\,\mathrm{mm}$ and $[0.22, 0.275]\,\mathrm{mm}$, respectively.

More details on the method will be given in the full paper. We will also apply the method to a magnetodynamic problem with hysteresis and discuss the choice of the elementary cells.

References

[1] O. de la Barrière, O. Bottauscio, M. Chiampi and A. Manzin, "A multiscale approach to predict classical losses in soft magnetic composites," *IEEE Trans. Magn.*, vol. 48, no. 4, 2012.

[2] O. Bottauscio, V. Chiado Piat, M. Chiampi, M. Codegone and A. Manzin, "Nonlinear homogenization technique for saturable soft magnetic composites," *IEEE Trans. Magn.*, vol. 44, no. 11, 2008.

[3] C. Cyr, P. Viarouge, S. Clénet and J. Cros, "Methodology to study the influence of the microstructure of soft magnetic composites on their global magnetization curve," *IEEE Trans. Magn.*, vol. 45, no. 03, 2009.

[4] I. Niyonzima, R. V. Sabariego, P. Dular and C. Geuzaine, "Computational homogenization for laminated ferromagnetic cores in magnetodynamics," *IEEE Trans. Magn.*, vol. 49, no 5, 2013.

[5] W. E, B. Engquist, X. Li, W. Ren and E. Vanden-Eijnden, "Heterogeneous multiscale methods: A review," *Comm. Comp. Phys.*, vol. 2, no. 3, pp. 367–450, 2007.

[6] A. Visintin, "Homogenization of doubly-nonlinear equations," *Rend. Lincei Mat. Appl.*, vol. 17, pp. 211–222, 2006.

[7] A. Visintin, "Scale-transformations of maximal monotone relations in view of homogenization," *Boll. Un. Mat. Ital.* (9) III (2010), 591–601.

[8] A. Bensoussan, J. L. Lions. and G. Papanicolaou, "Asymptotic Analysis for Periodic Structures," *American Mathematical Society*, 2011.