An Accurate Mesh Based Equivalent Circuit Approach to Thermal Modelling

Nick Simpson, Rafal Wrobel, Phil H. Mellor
University of Bristol, Bristol, BS8 1TR, UK
nick.simpson@bristol.ac.uk

***Abstract*—This paper presents a method of automatically constructing and parameterising accurate lumped parameter thermal equivalent circuits with nodes arranged in a regular mesh pattern. The approach exhibits a number of advantages over conventional lumped element thermal equivalent circuits including superior thermal field resolution, reduced model construction and setup times and more accurate identification of hot-spot temperatures and their location. The method serves as a desirable compromise between the fine detail and high computational cost of a full finite element analysis and the coarse detail and short solution times afforded by lumped element thermal equivalent circuits.**

***Index Terms*—Lumped parameter, equivalent circuit, automated construction, cylindrical thermal element**

I. INTRODUCTION

Finite Element Analysis (FEA) and Computational Fluid Dynamics (CFD) are widely adopted numerical tools for the detailed thermal analysis of electrical machines and devices. However the computational cost, particularly for transient analyses, makes FEA and CFD unfavourable modelling options for inclusion within iterative design procedures and optimisation [1]. An accepted alternative is to formulate lumped parameter Thermal Equivalent Circuits (TECs) where the nodal voltages represent temperatures of interest within the device, [2], [3]. These models are often manually constructed based on experience and as such contain as few nodes as possible which afford very short solution times but compromise thermal field detail and solution accuracy (if uncalibrated). Consequently they tend not to accurately capture certain temperatures of interest such as peak winding temperatures, Fig. 1.

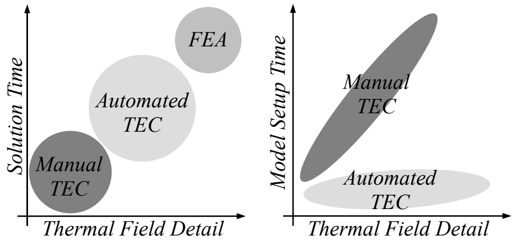

Fig. 1. Left: A qualitative comparison between thermal field detail and solution time. Right: A qualitative comparison of model setup time and thermal field detail.

A method of using a geometric and material description of a device to automatically construct an accurate TEC with nodes arranged in a regular mesh is presented. The resultant hybrid approach maintains short solution times while allowing superior thermal field resolution, reduced model construction and setup times and allows the identification of hot-spot temperatures and their location.

II. CASE STUDY

The method is demonstrated by the transient thermal analysis of a linear actuator stator which is modelled axisymmetrically in two-dimensions, Fig. 2, with material properties given in [4]. A fixed power is applied to the stator windings over a 15 second interval in accordance with Fig. 5. Adiabatic boundary conditions are applied to each surface except the outer cylindrical surface which is defined as a convection boundary with a heat transfer coefficient of $20W/m^2.K$. Interface thermal resistance is neglected.

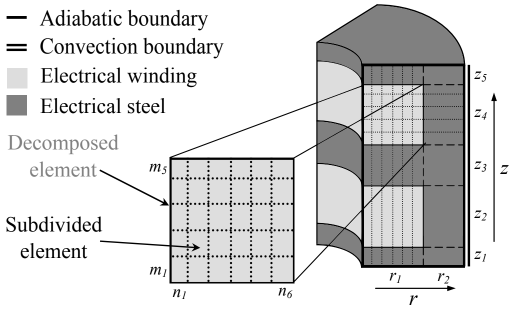

Fig. 2. Tubular linear actuator stator (segment shown) detailing the cross-sectional decomposition, subdivision and imposed boundary conditions.

III. AUTOMATED TEC GENERATION AND EVALUATION

The proposed method consists of three steps:

A. Geometry Decomposition

Referring to Fig. 2, the stator cross-section is systematically decomposed into a number of regular quadrilateral elements $(r_1, z_1) \rightarrow (r_2, z_5)$ referred to as *decomposed elements*, which are further subdivided into $(m_1, n_1) \rightarrow (m_n, n_n)$ elements referred to as *subdivided elements*.

B. TEC Generation

1) General Cylindrical Element: Each element within the TEC model is represented by a general cylindrical element, Fig. 3, which is formulated using T-networks to report the average temperature over the volume at the central node, $\overline{T}$ and accurately accounts for radial and axial heat flow and internal heat generation, [5]. Since separate T-networks represent the axial and radial axes, material anisotropy can easily be accounted for.

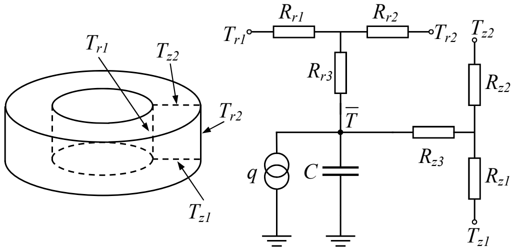

Fig. 3. T-network equivalent circuit representing the axial and radial heat flow through a cylinder.

2) Netlist Generation: The cylindrical element sub-circuits are connected together in a netlist and parameterised using geometric and material information. Resistors and voltage sources are added to represent the surface boundary conditions and ambient temperature conditions respectively. A time-varying current source is added to supply the winding power. The resulting netlist is output to a SPICE compatible file.

C. TEC Evaluation

The SPICE netlist is passed to and evaluated by NGSPICE-24. The relevant nodal voltages $\left(\overline{T}\right)$ of each cylindrical element are extracted from the solution.

IV. RESULTS

The automatically generated TEC is validated using a benchmark transient FEA (FEMM 4.2) model using a fine mesh and a small time-step (4ms) to ensure solution accuracy (32.5 minute solution time). The global percentage error of the TEC is defined as the difference between the benchmark FEA solution and the TEC solution as a percentage, averaged over each TEC element and each time-step, Fig. 4. The solution time is defined as the time taken to read the TEC netlist file, evaluate the transient response and process the resulting nodal voltages. Fig. 4 illustrates the relationship between the number of TEC elements, the global percentage error and the solution time. As expected, the model accuracy increases rapidly as a function of the number of elements owing to a more accurate representation of the heat flow throughout the stator, however the solution accuracy converges to a minimum error $(\sim 0.5\%)$. This minimum error is imposed by the assumptions made in the mathematical formulation of the cylindrical element.

The average temperatures over the *decomposed elements* are plotted over time for the benchmark FEA and a TEC model along with the applied winding power. The TEC model shown in the figure has a solution time of 4.7 seconds, Fig. 4, contains 160 elements and has an average global percentage error of 1.2% which is evidenced by the temperature traces, Fig. 5.

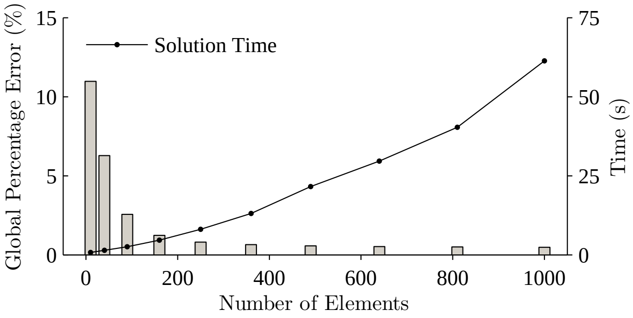

Fig. 4. Average percentage error compared to the FEA benchmark solution along with solution time.

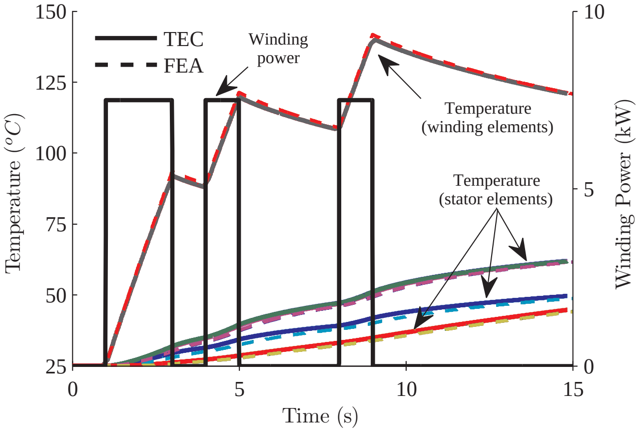

Fig. 5. Average decomposed element temperatures for FEA and automated TEC models with winding power duty cycle.

V. CONCLUSION

A method of automatically generating TECs for simple geometries with cylindrical symmetry is presented and shown to yield accurate results with minimal computational cost when compared with a more conventional FEA approach. The accuracy and solution time of the TEC is a function of the number of elements. The automated TECs significantly reduce model setup and construction time when compared with conventional, manually constructed, TECs and yield accurate results while maintaining short solution times. The method is suitable for extension into the third dimension and to more complex geometries (including curved edges) requiring further generalised elements to be developed [4]. The method allows accurate TECs to be rapidly constructed and solved and as such, machine geometry can become a design variable within iterative design and optimisation routines.

REFERENCES

[1] P. Vong and D. Rodger, "Coupled electromagnetic-thermal modeling of electrical machines," *Magnetics, IEEE Transactions on*, vol. 39, no. 3, pp. 1614 – 1617, may 2003.

[2] J. Fan, C. Zhang, Z. Wang, Y. Dong, C. Nino, A. Tariq, and E. Strangas, "Thermal analysis of permanent magnet motor for the electric vehicle application considering driving duty cycle," *Magnetics, IEEE Transactions on*, vol. 46, no. 6, pp. 2493 –2496, june 2010.

[3] N. Rostami, M. feyzi, J. Pyrhonen, A. Parviainen, and M. Niemela, "Lumped-parameter thermal model for axial flux permanent magnet machines," *Magnetics, IEEE Transactions on*, vol. PP, no. 99, p. 1, 2012.

[4] R. Wrobel and P. Mellor, "A general cuboidal element for three-dimensional thermal modelling," *Magnetics, IEEE Transactions on*, vol. 46, no. 8, pp. 3197 –3200, aug. 2010.

[5] P. Mellor, D. Roberts, and D. Turner, "Lumped parameter thermal model for electrical machines of tefc design," *Electric Power Applications, IEE Proceedings B*, vol. 138, no. 5, pp. 205 –218, sep 1991.