Mixing of liquids with a rotating current density

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Abstract—**Numerical tools for the study of mixing of small quantities of liquid using a rotating current density and a fixed magnetic field are developed and analyzed in this paper.**

Index Terms—**Magnetohydrodynamics, Electromagnetic forces, Current density, Permanent magnets**

I. INTRODUCTION

In many biomedical applications very small volumes of liquid, usually of the order of $1ml$, have to be mixed in order to obtain the desired effect (mixing or reaction). Species transport by molecular diffusion is very slow and turbulent agitation is not a viable solution since it leads to the breakup of the long fragile molecules in the liquid. If the liquid is slightly conductive, as is the case in many biological liquids, the Lorentz body force can be used to propel the liquid from one region to another. Such a device has already been employed by [1] but there is room for improvement from an electrical engineering point of view. Here we first describe and evaluate a device with a rotating current density field similar to the rotating magnetic field of electrical motors. The second purpose is to develop the necessary numerical tools to study the mixing process both qualitatively and quantitatively.

II. DEVICE AND SHORT DESCRIPTION OF THE MODELS

Six electrodes $n = 1, \ldots, 6$ are placed in the conducting liquid's tank at positions $R\ (\cos{(n\,\pi/6)}\ \vec{k}_x + \sin{(n\,\pi/6)}\ \vec{k}_y)$ and their potentials e_n are driven such that:

$$e_n = \Re\{\underline{E}\ \exp^{j\,(\omega\,t + n\,\pi/6)}\} \tag{1}$$

where $\underline{E}$ is the complex amplitude of the applied voltage and ω is the pulsation (see Fig. 1). The resulting current density in the conducting liquid is then approximatively uniform in amplitude j_0 and has a rotating direction

$$\vec{j} = j_0\ (\cos{(\omega\,t)}\ \vec{k}_x + \sin{(\omega\,t)}\ \vec{k}_y) \tag{2}$$

This is the 2 pole rotating current density.

Simultaneously a magnet is placed just under the center of the cylindrical tank; the magnetic field h_0 just above it (region D) is approximatively uniform. The Lorentz force density is approximatively:

$$\vec{f} = \begin{cases} \mu_0\ h_0\ j_0\ (\sin{(\omega\,t)}\ \vec{k}_x - \cos{(\omega\,t)}\ \vec{k}_y)\ \text{in}\ D \\ \vec{0}\quad \text{elsewhere} \end{cases} \tag{3}$$

When the pulsation ω is zero, the direction of the force density is fixed and the generated fluid flow corresponds, approximatively, to a dipolar stream function. A particle placed
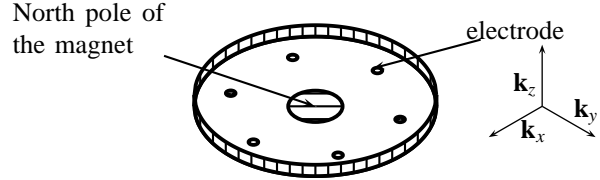

Figure 1: Geometry of the tank, the electrodes and the magnet.

anywhere describes a certain closed path and returns to its initial position in a time $2\,\pi/\Omega$.

When the pulsation ω does not vanish, with a steady-state approximation where the change of fluid flow responds instantaneously to the change of force density, the particle is subject to a motion driven by two pulsations ω and Ω.

III. NUMERICAL MODEL

The above description does not take into account the details of the multi-physics modelling, which are summarized in this section.

Four fields are in presence : the current density, the magnetic field due to the magnet (which produces the Lorentz force density), the velocity field and the concentration of chemical species (of positions of the particles).

The current density $\vec{j}$ and the velocity field $\vec{v}$ are numerically computed on the same tetrahedral mesh as in the 3D geometry. The magnetic field is computed by the Biot and Savart equation with a Coulombian magnet description.

The electrical potential and P1 finite elements are used for the current density, this does not lead to too many difficulties. The magnetic field is needed outside of the poles of the magnet where the Biot and Savart formula does not have any singularity, then a numerical integration (the poles are triangularly meshed and a Gauss-Legendre quadrature is used) is suitable. The magnetic field is then computed on the center of each tetrahedron and the Lorentz force is obtained as a vector constant for each element (P0).

The Eulerian velocity $\vec{v}$ and pressure P fields are calculated by solving the Stokes equations since the inertial terms can be neglected due to the low velocities. This Stokes problem has a variational formulation where the velocity minimizes a functional (the power corresponding to the viscous dissipation) under the zero divergence constraint where pressure is the Lagrange multiplier. It is a saddle point problem for which the Uzawa algorithm is suitable [2]. The velocity $\vec{v}$ is split
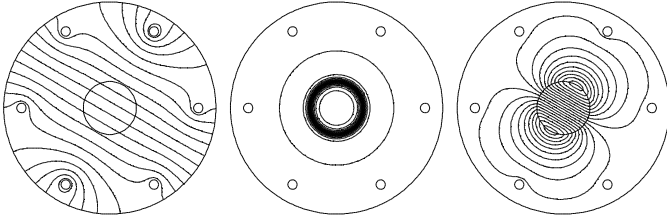
Figure 2: The electrical potential (left), the vertical component of the magnetic field (middle), and the lines of force (right) on the surface D_∂ of the electrolyte.
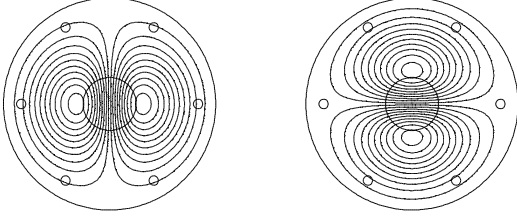

Figure 3: On the surface D_θ: Streamlines when the electrical current density is vertical (left) and horizontal (right).

into its components (u, v, w) each being approximated $P2$ on the mesh, the pressure P being approximated $P1$.

We restrict the analysis of advection to particles remaining on the surface of the electrolyte; then the triangular 2D submesh of this surface is extracted and only the trace of the velocity field $\vec{v}_\partial = u\,\vec{k}_x + v\,\vec{k}_y$ is used. Due to the boundary conditions on this free surface, w and its normal derivative are zero, the 2D velocity field $\vec{v}_\partial$ is thus divergence free and can be formulated as $\vec{v}_\partial = \vec{\nabla} \times (\psi\,\vec{k}_z)$ where ψ, the stream function, is computed on the 2D-mesh with a P1 approximation.

A set of N particles, initially at the positions $x_n\,\vec{k}_x + y_n\,\vec{k}_y$ $n = 1\dots N$, have trajectories $X_n(t)\,\vec{k}_x + Y_n(t)\,\vec{k}_y$ which are solutions of

$$\frac{dX_n}{dt} = -\partial_y\psi(t, X_n, Y_n) \quad ; \quad \frac{dY_n}{dt} = -\partial_x\psi(t, X_n, Y_n). \qquad (4)$$

This Hamiltonian system depends explicitly on time but with a known law. One can then obtain an exact integration from one edge of an element to the other one.

If the concentration is fixed as equal to unity inside a moving region D_t and zero outside, one only requires to consider the advection of its boundary ∂D_t, the material points composing such a boundary can be moved as particles (Lagrangian tracking).

Freefem++ is used for the actual calculations ; it is effective for most couplings and is open enough so that it is possible to write scripts operating at the level of finite elements such as the advection of particles.

IV. RESULTS AND DISCUSSION

The electrical potential, the vertical component of the magnetic field and the lines of force are plotted on Fig. 2. At a given instant of time, the electrical potential (Fig. 2 left) shows that the electrical current density is almost straight
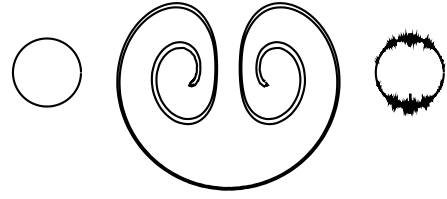

Figure 4: A circular blob of dye submitted to a force directed first downwards and then upwards.

Figure 5: Blob of dye after (from left to right) 1, 2, and 5 periods with the rotating field and $2\,\pi/\omega = 2/3\ Tm$.

in the region where the magnetic field is important (Fig. 2 middle). The Lorentz force density is represented by its lines of force (Fig. 2 right) i.e. the iso-ξ where ξ minimizes $\int_{D_\partial}(\vec{j}\times\vec{b} - \vec{\nabla}\times(\xi\,s\vec{k}_z))^2\,d\vec{x}^2$, they are drawn on the surface D_∂ of the liquid region.

The velocity on D_∂ is represented by the stream lines (Fig. 3), for the two orientations (vertical and horizontal) of the Lorentz force density.

In a first test, the advection of a blob of same radius as the magnet (Fig. 4 left) and with only the vertical lines of force is considered. Once its perimeter has increased to about 10 times its original value (Fig. 4, middle), the opposite lines of force are applied and the initial blob is obtained with a little fuzziness (Fig. 4, right) due to numerical diffusion, which can be estimated. The difference between initial and final blobs is 4% root mean square and 20% maximum.

Finally the spiral shape of a blob of dye after 1, 2 and 10 periods $2\pi/\omega$ when the rotating field is applied is shown in Fig. 5. If $T_m = 2\pi/\Omega$ is the mean period of the closed path of streamlines (Fig. 3) ; $2\pi/\Omega$ if chosen such as $2/3\ T_m$. A Comparison between blob of dyes obtained at the same time but with different ratios between ω and Ω is shown Fig. 6.

REFERENCES

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Figure 6: Comparison of blobs of dye at the same time between $2\,\pi/\omega = 2/3\ Tm$ (left), $1/3\ Tm$ (middle) and $2/9\ Tm$ (right).