# A General Arc-Segment Element for Three-Dimensional Thermal Modelling

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***Abstract*—This short paper introduces a lumped parameter thermal equivalent circuit representation of a generalised arc-segment that can be used in the accurate three-dimensional thermal modelling of transformers, wound passive components and electrical machine geometry, particularly stator/rotor teeth and end-windings. Commonly used two-resistor lumped parameter thermal networks do not accurately account for internal heat generation, which is essential for accurate temperature prediction. In contrast, the general arc-segment element proposed in this paper uses a T-network formulation and caters for internal heat generation along with material anisotropy. The element is verified over a range of arc-angles and outer/inner radius-ratios using two and three-dimensional finite element analysis showing good agreement.**

***Index Terms*—Lumped parameter, three-dimensional thermal equivalent circuit, general arc-segment**

## I. INTRODUCTION

A major limiting factor on the performance of electrical machines and devices is their ability to dissipate generated heat. As such, accurate thermal modelling of electrical devices is very desirable and is typically achieved using analytical, lumped parameter or numerical methods. Frequently employed techniques include lumped parameter Thermal Equivalent Circuits (TECs), Finite Element Analysis (FEA) and Computational Fluid Dynamics (CFD). Numerical FEA and CFD methods provide fine thermal field detail but suffer from long model setup and solution times. Lumped parameter TECs offer short solution times and acceptable thermal field detail which makes it a valuable tool, however, the correct formulation of the equivalent circuit is essential to provide accurate temperature prediction.

The two-resistor element commonly employed in thermal modelling is derived from the one-dimensional steady-state heat diffusion equation with zero internal heat generation and is shown to yield incorrect temperature predictions in the presence of internal heat generation [1], [2]. To overcome this inaccuracy, three-dimensional TEC modelling methods based on the use of accurately formulated general cylindrical elements and general cuboidal elements have been proposed [2], [3] and shown to yield accurate results. However, the device geometries that can be represented by cylindrical and cuboidal elements are limited. This paper presents the development of a general arc-segment element, Fig. 1, which allows many more geometries to be modelled with TECs and is particularly relevant to the modelling of stator/rotor teeth and end-windings. The general arc-segment element is verified using both two and three-dimensional FEA over a range of outer/inner radius-ratios and arc-angles.

## II. MATHEMATICAL DESCRIPTION

Fig. 1 illustrates an arc-segment described by the arc-angle $\alpha$ (degrees), inner radius $r_1$, outer radius $r_2$ and axial length $l_a$. The heat flow in the radial, circumferential and axial directions are assumed to be independent. Each axis is represented by a one-dimensional T-network, [4] allowing the thermal conductivity to be set in each axis. A T-network is necessary to provide an accurate prediction of the average temperature of the element in the presence of heat generation. The T-networks join at a central node representing the average temperature over the volume, $\overline{T}$. The current source and capacitor represent internal heat generation and thermal storage respectively.

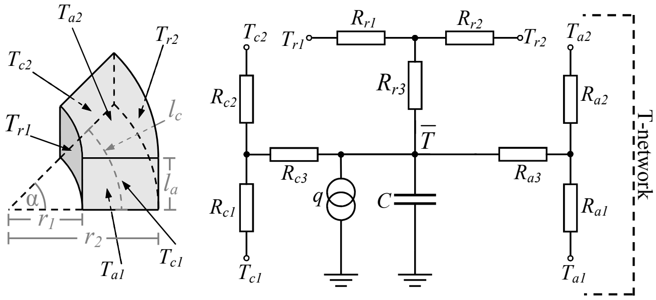


Fig. 1. Geometric and T-network representation of an arc-segment

The thermal resistances in each T-network are derived through superposition by evaluating two cases of the one dimensional steady state heat diffusion equation [2], [4]:

- Zero internal heat generation
- Zero surface temperatures with internal heat generation

The cases are evaluated in Cartesian coordinates [2] for axial and circumferential heat flow and radial coordinates [4] for the radial heat flow. Equation (1) and (2) representing axial heat flow are modified from [2], substituting the arc cross-sectional area. Equation (3) and (4) representing circumferential flow are adapted from [2] where the length term is replaced by an average arc-length term, $l_c$. Equations (5) to (8) representing radial heat flow are modified from [4] to represent a partial cylinder, this is achieved by scaling the terms by the arc-length, $l_c$.

### A. Axial Heat Flow

$$R_{a1} = R_{a2} = \frac{180 l_a}{\alpha \pi k_a \left( {r_2}^2 - {r_1}^2 \right)} \tag{1}$$

$$R_{a3} = -\frac{60 l_a}{\alpha \pi k_a \left( {r_2}^2 - {r_1}^2 \right)} \tag{2}$$

### B. Circumferential Heat Flow

$$l_c = \frac{\alpha}{360}\pi\left(r_1 + r_2\right) \qquad R_{c1} = R_{c2} = \frac{l_c}{2k_c l_a\left(r_2 - r_1\right)} \tag{3}$$

$$R_{c3} = -\frac{l_c}{6k_c l_a\left(r_2 - r_1\right)} \tag{4}$$

### C. Radial Heat Flow

$$R_{r1} = \frac{90}{\alpha\pi k_r l_a}\left[\frac{2{r_2}^2 ln\left(\frac{r_2}{r_1}\right)}{{r_2}^2 - {r_1}^2} - 1\right] \tag{5}$$

$$R_{r2} = \frac{90}{\alpha\pi k_r l_a}\left[1 - \frac{2{r_1}^2 ln\left(\frac{r_2}{r_1}\right)}{{r_2}^2 - {r_1}^2}\right] \tag{6}$$

$$R_{r3} = K\left[{r_2}^2 + {r_1}^2 - \frac{4{r_2}^2{r_1}^2 ln\left(\frac{r_2}{r_1}\right)}{{r_2}^2 - {r_1}^2}\right] \tag{7}$$

Where:

$$K = \frac{-45}{\alpha\pi k_r l_a\left({r_2}^2 - {r_1}^2\right)} \tag{8}$$

## III. Model Validation

### A. Two-dimensional Case Study

Here the two-dimensional radial and circumferential heat flow network (3) to (8), which is approximated by two one-dimensional T-networks is tested. In order to establish the validity of the approximation an arc-segment is modelled using two-dimensional FEA with a fixed volumetric heat generation, isotropic thermal conductivity, $(10W/m.K)$ and convective boundary conditions on each side, $(25W/m^2.K)$. The average temperature is calculated as a function of arc-angle $\alpha$ and the ratio of outer radius $r_2$ to inner radius $r_1$. The percentage difference in average temperature reported by the FEA and the general arc-segment element model is plotted in Fig. 2.

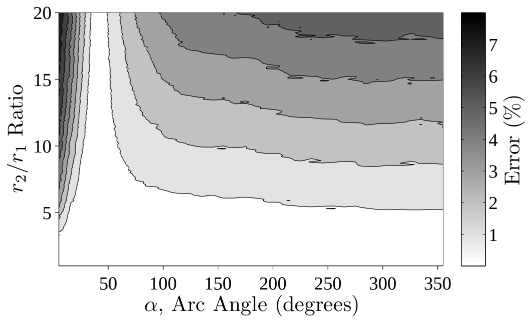


Fig. 2. Comparison between average temperatures reported by 2-dimensional FEA and TEC. The percentage difference in reported temperature is plotted as a function of arc-angle and the ratio of the outer radius to the inner radius.

It is shown, Fig. 2, that for large ratios of outer to inner radius and small arc-angles the arc-segment element can overpredict the average temperature by up to $7\%$, similarly, for large radius-ratios and large arc-angles the arc-segment element is up to $5\%$ in error. However, for cases where the outer to inner ratio is $4$ or less, the arc-segment element is in less than $1\%$ error irrespective of arc-angle, which is a region highly applicable to electrical machine design.

TABLE I
ARC-SEGMENT TEST CASE PROPERTIES

| | Convection Boundary $(W/m^2.K)^a$ | | | | | | Heat Source $(W)$ |
|---|---|---|---|---|---|---|---|
| *Case* | $h_{r1}$ | $h_{r2}$ | $h_{a1}$ | $h_{a2}$ | $h_{c1}$ | $h_{c2}$ | $q$ |
| 1 | 25 | 25 | 25 | 25 | 25 | 25 | 5 |
| 2 | 25 | 25 | 25 | 25 | 50 | 25 | 10 |

$^a T_{ambient} = 75^oC$, $k_r = 2$, $k_a = 2$, $k_c = 200$ $(W/m.K)$

### B. Three-dimensional Case Study

In order to test the validity of the general arc-segment element in three-dimensions the temperature predictions from a three-dimensional FEA model of an arc-segment, Fig. 3, are compared with those predicted by a general arc-segment element. Two simple cases detailed in Table I are evaluated, with the results compared in Table II.

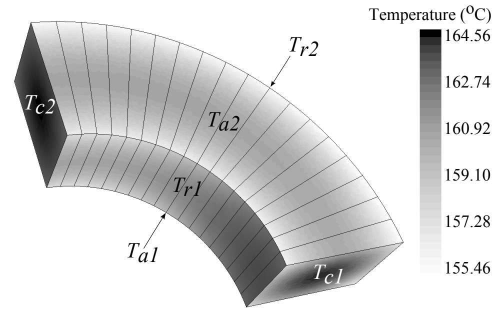


Fig. 3. 3-dimensional FEA temperature solution for an arc-segment with $r_1 = 0.025$, $r_2 = 0.045$, $l_a = 0.15$ and $\alpha = 90$. The prescribed boundary conditions and internal heat generation for two test cases are given in Table I.

TABLE II
ARC-SEGMENT TEST CASE RESULTS

| *Case* | $\overline{T}$ FEA $(^oC)$ | $\overline{T}$ TEC $(^oC)$ | Error (%) |
|---|---|---|---|
| 1 | 120.8 | 121.4 | 0.4 |
| 2 | 160.8 | 161.7 | 0.6 |

For these simple cases the general arc-segment element shows close agreement with the three-dimensional FEA model which verifies the axial T-network. This result combined with the two-dimensional analysis suggests that the developed general arc-segment element is valid, however, more complex test cases are required to fully justify the formulation.

## IV. Conclusion

A general lumped parameter TEC element representing an arc-segment has been developed which caters for internal heat generation and material anisotropy. The element has been verified using two and three-dimensional FEA with encouraging results particularly over a range of arc-angles and radius-ratios applicable to electromagnetic device design.

## References

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