Numerical and Experimental Validation of Discharge Current with Generalized Energy Method and Integral Ohm's Law in Dielectric Liquid Media

Ho-Young Lee[1], Jae-Seung Jung[1], Hong-Kyu Kim[2], Il-Han Park[3], and Se-Hee Lee[1]
[1]Department of Electrical Engineering, Kyungpook National University, Daegu, 702-701, Korea
[2]Korea Electrotechnology Research Institute, Changwon, 642-120, Korea
[3]College of Information and Communication Engineering, Sungkyunkwan University, Suwon, 440-746, Korea
shlees@knu.ac.kr

***Abstract*—Discharge currents were evaluated and compared with one another by using the Sato's equation, the generalized energy method, and the integral Ohm's law in dielectric liquid media. Additionally, to verify the numerical results, an experimental setup was proposed with a multi-port system composed of a tip and separated conducting shells. The numerical results will be compared with those from the experiments which will be the first trial in dielectric liquid with a multi-port system.**

***Index Terms*—Discharge current, energy method, integral Ohm's law, Poynting's theorem, Finite Element Method (FEM).**

I. INTRODUCTION

Calculation of the current flowing between any two electrodes during discharge simulation is important because the terminal current is often one of the only parameters that can be measured during actual breakdown testing. In high voltage discharge and plasma analysis, the expanded Sato's equation has been widely used for calculating a terminal current [1]-[2]. The final Sato's equation can be successfully applied to most of discharge problems. As in the arc simulation, however, when the magnetic field effect is significant, the calculating method for terminal current should be modified and generalized.

With the energy balance equation, here, we tested a generalized method to calculate a terminal current by directly adopting Poynting's theorem incorporating the Finite Element Method (FEM) which has been successfully applied to discharge analysis. This generalized energy method naturally covers the time-varying voltage sources, magnetic field effect, and any dielectric media such as gas, liquid, and solid.

As an alternative method, we also tested the integral Ohm's law, which can be applied to multi-port systems. The Sato's equation and the generalized energy method, however, can be applied to only a two-terminal electromagnetic system because it is difficult to separate the energy contributions for each port. To test above two methods, the results were compared to those from the Sato's equation and an analytic solution [3]. Finally, the numerical results from the Sato's equation, the generalized energy method, and the integral Ohm's law will be compared with those from the experimental setup of which we proposed in this paper.

II. EXPANDED SATO'S EQUATION

The expanded Sato's equation considering three carriers and a time-dependent applied voltage was expressed as [2]

$$I = \frac{1}{V_a}\int_\Omega \mathbf{J}_c \cdot \mathbf{E}_L dv + \frac{\varepsilon_0}{V_a}\int_\Omega \frac{\partial \mathbf{E}_L}{\partial t}\cdot \mathbf{E}_L dv \tag{1}$$

with $\mathbf{J}_c = (N_p\mathbf{V}_p - N_e\mathbf{V}_e - N_n\mathbf{V}_n - D_p\nabla N_p + D_e\nabla N_e + D_n\nabla N_n)$

where I is the external circuit current, V_a the applied voltage, $\mathbf{J}_c$ the conduction current, $\mathbf{E}_L$ the electric field intensity from Laplace's equation, ε_o the dielectric permittivity in air, and $\int_\Omega dv$ a volume integral over the discharge space.

III. INTEGRAL OHM'S LAW AND GENERALIZED ENERGY METHOD

In general the terminal current is due to conduction current in a dielectric medium as well as displacement current due to the time rate of change of the surface charge on the electrodes as

$$I = \int_S \left(\mathbf{J}_c + \varepsilon\frac{\partial \mathbf{E}}{\partial t}\right)\cdot d\mathbf{a} \tag{2}$$

where $\mathbf{E}$ is the total electric field intensity due to the applied voltage and space charge distributions, and (2) is known as the integral's Ohm's law.

By considering any general field within a volume, one realizes that the energy contained by that field must be distributed throughout space with a local energy density W at every point in the volume. With field generalizations and quasi-static approximations, the power flowing into a volume Ω, enclosed by the surface S_a, can be expressed as

$$\sum_{i=1}^{n} V_i I_i = \frac{d}{dt}\int_\Omega W dv + \int_v P_d dv \tag{3}$$

with $W = (1/2)\varepsilon\mathbf{E}\cdot\mathbf{E} + (1/2)\mu\mathbf{H}\cdot\mathbf{H}$ and $P_d = \mathbf{E}\cdot\mathbf{J}_c$

where V_i is an applied voltage at a terminal, I_i the corresponding current at that terminal, μ the magnetic permeability, and $\mathbf{H}$ the magnetic field intensity.

Generally, the FEM gives more accurate global quantities such as energy and total power dissipation because the procedure of FEM follows the global energy minimization condition. This is based on the principle of virtual work which is incorporating with the variational formulation of the Ritz method for deriving the FEM formulation in static and transient cases. This generalized energy method, therefore, is harmony well with the FEM.

It is important to point out that it is only possible to use (1) and (3) to establish the current flowing into a volume when the number of terminal pair n=1, i.e., when the volume only has two terminals. On the contrary, the integral Ohm's law, (2), can be applied to a multi-port system when n>1.

IV. NUMERICAL ANALYSIS RESULTS

A. Verification of Numerical Setup with Parallel Plane Model

To verify our numerical setup and compare with each other, first, we simplified our governing equations to one carrier system in plane-plane 2-D XY geometry. As we can see in Fig. 1, the energy approaches, Sato's equation and generalized energy method, produced almost the same results as analytic solutions [3]. Even though the integral Ohm's law has an ability of measuring the terminal current on each electrode, it contained some numerical distortions where the value of space charge was high.

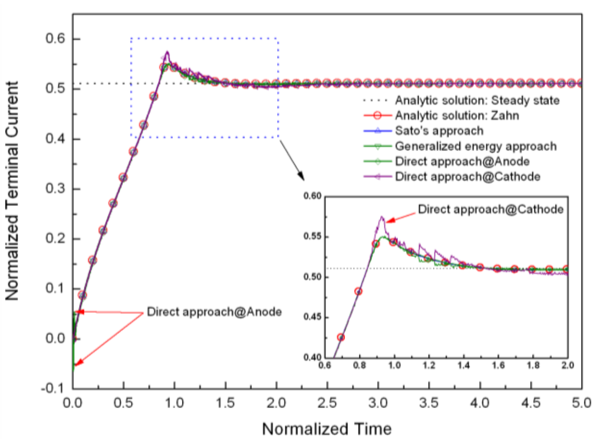

Fig. 1. Terminal current profiles from the various methods. The analytic solution was evaluated by Zahn's approach in [3]. Here, the direct approach represents the integral Ohm's law.

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B. Experimental Validation for a Multi-port System with Tip-Ring Electrodes in Dielectric Liquid

Figs. 2-4 show the experimental setup and preliminary numerical results with a high step voltage input. The summation of each shell current by the integral Ohm's law agreed well with that by the generalized energy method. In the extended paper, we will show those results with experiments in detail.

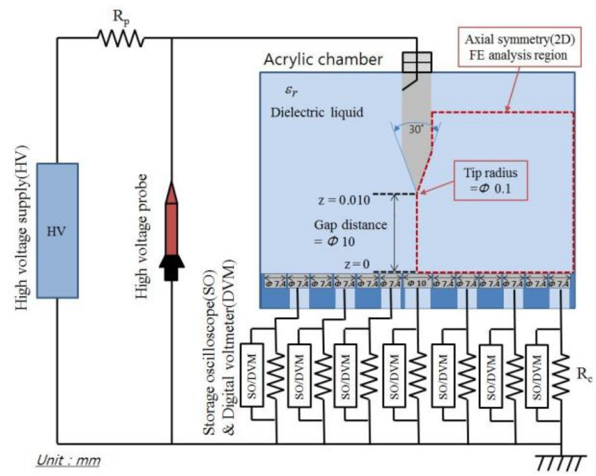

Fig. 2. Schematic representation of a set-up for experimental and numerical validation with tip-plates model for measurement of discharge current.

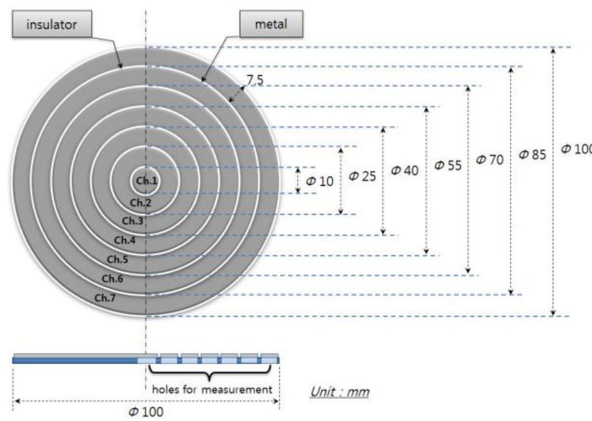

Fig. 3. Plate electrodes for current measurement.

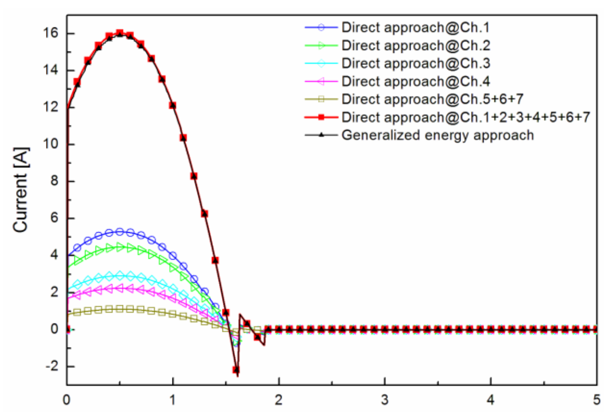

Fig. 4. Calculated current profile with high voltage input by the integral Ohm's law (direct approach) and generalized energy method for tip-plates model. Total current by the direct approach was the same as that by the generalized energy method. The numerical setup for discharge analysis was reported in [4] and [5] in detail.