On the Shoulders of Giants

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***Abstract*—The paper reviews the main achievements and presents a list of the principal researchers in computational electromagnetics.**

***Index Terms*—Computational electromagnetics.**

I. Introduction

In this paper we attempt to list the principal pioneers in computational electromagnetics. As Isaac Newton remarked, *if I have seen a little further it is by standing on the shoulders of Giants*, so we will attempt to name the key researchers that have made significant advances in our subject by examining historical published work. We wish to build on the ideas first discussed at Compumag Shenyang [1], further developed and reported at Compumag Sydney [2] and CEM Brighton [3].

As proposed before [1], [2] and [3], to make such a list we need to define a criterion for inclusion. No single published paper, of course, is completely new but it must not merely duplicate earlier work and must contain at least one innovative step. A significant step forward is valid however if it improves the efficiency and applicability of an existing method or indeed adapts a technique previously applied to a different discipline say from a branch of mathematics.

We are only concerned here with developments in computational electromagnetics (CEM) in which the problems arising in the research and development of scientific and engineering devices are today solved using advanced digital computers. This subject is closely coupled to the major new discoveries made by the great founding fathers of the science of electromagnetic fields, e.g. Michael Faraday, George Green, Clerk Maxwell, amongst many others, they are self-evidently 'giants' of the subject also, but their work and legacy is outside our remit here as they belong to the pre-computer age. However, there is an overlapping period in which numerical mathematics and analogue methods were used, which ultimately led to the development of algorithms that later proved to be extremely successful and appropriate for digital computers.

II. Finite Difference Methods (FD)

In Munich 1892/93 Ludwig Boltzmann (1844-1906) gave a course on the mechanical potential in which he derived the 'unit square' (5 point) formula which after his tragic death was reported by his student, the astronomer Hugo Buchholz. In Table I we give the subsequent major developments and attribution.

III. Finite Element Method (FE)

Origins of the method lie in applying variational methods to the solution of problems of equilibrium and vibrations by Courant (1943) [14] but also on work of Rayleigh (1870), Ritz (1909) and Galerkin (1915) [15]. Method was named and made systematic by Turner, Clough et al in 1956 [16] and used effectively in the aircraft industry. An influential early pioneer was Zienkiewicz (1967) [17] who became the principal evangelist for the method by recognising its potential for generalisation and for modelling complex structures. The Mathematical basis was established by Oden (1969) [18].

TABLE I
CONTRIBUTORS TO THE EVOLUTION OF THE FINITE DIFFERENCE METHOD

Date	Name	Comments	Ref
1908	C.D.T.Runge	Unit Square formula simple problems without proof	[5]
1910	L F Richardson	First systematic attack with proofs and real applications	[6]
1918	H Liebmann	Extensions and citing Boltzmann's contribution	[4]
1935	R.V. Southwell	Relaxation Method	[7]
1962	F.C. Trutt	Low Frequency	[8]
1966	E Erdelyi et al	Low Frequency	[9]
1978	A Viviani et al	Low Frequency	[10]
1976	W Muller et al	Low Frequency	[11]
1966	K Yee	High Frequency	[12]
1977	T Weiland	High & Low Frequency	[13]

An early implementation was carried out by Alan Winslow in 1964, 1967 [19]; his TRIM code was widely used for electromagnetic problems. He demonstrated the equivalence of FD, FE & resistor analogue for solving the Poisson Equation discretised by an irregular triangular mesh. The first implementation for the design of Electrical Machines was introduced by Silvester and Chari [20]. Subsequent extensions in the machine modelling were made by C J Carpenter, J.L. Coulomb, A. Konrad and J.C. Sabonnadiere [21], [22]. The method was further extended to 3D by J. Simkin and C. W. Trowbridge [23].

IV. Integral Methods (IM)

Integral methods, unlike FD & FE use integral equation forms of the field equations, also known as Moment Methods which were described theoretically by Harrington in 1968 [24]. Also in 1968 A. Halacsy implemented the moment method based on a point dipole [25]. This approach was generalised to include interactive 3D modelling and non-linear materials in the UK in 1972 [26].

Another class of integral procedures is the so called Boundary Element Method [27], [28], [29] based on applications of Greens integral theorems. Whilst these methods are often difficult to apply they can produce accurate economic solutions.

V. Outstanding Achievements

There have been many outstanding contributions in more recent times but the lack of space available to us means that only brief mention can be given. However, in the final presented version, a fuller account and attribution will be discussed. See Table II for a list of topics and some of the principal researchers.

TABLE II
RECENT CONTRIBUTORS TO CEM

Topic	**Principal Researcher**	**Reference**
ICCG Method	J.A. Meijerink and V. der Vorst	[30]
Delaunay Meshing	Z. Cendes et al	[31]
Kelvin Transformation	E.M. Freeman and D.A. Lowther	[32]
Edge Elements	A. Bossavit	[33]
Dual Energy Methods	P. Hammond and J. K. Sykulski	[34]
Tubes and Slices	P. Hammond and J. K. Sykulski	[34]
Material Modelling	D.C. Jiles and D.L. Atherton	[35]
Forces	J.L. Coulomb and G. Meunier	[36]
Motion	D. Rodger, H Lai and P Leonard	[37]
Fast Multipole	L. Greengard and V. Rokhlin	[38]
Finite Integration	A. Bossavit and L. Kettunen	[39]

VI. General Comments

The motto of this paper has a special meaning, suggesting using the understanding gained by major thinkers who have gone before in order to make intellectual progress. The phrase has been used by many, including the title of a fairly recent book written by Stephen Hawking [40], who shows how each of his selected world's great thinkers built upon the genius of his predecessors. Although the achievements in computational electromagnetics over the years may not have been as spectacular as some of the discoveries described in this book, the underlying principle is the same and as a community we must both acknowledge the accomplishments and learn from the lessons of the past.

This is the fourth attempt on our part to induce discussion and provide a forum for proper debate. The readers are invited and encouraged to visit the International Compumag Society's web page [41] for more information and relevant links. It is indeed – in our view – one of the important roles of the Society to conduct such debate.

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