

Two-Scale Homogenization of the Nonlinear Eddy Current Problem with FEM

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Abstract—An efficient and accurate computation of the eddy current losses in laminated iron cores of electric devices is of great interest. Modeling each laminate individually by the finite element method requires many elements and leads to large systems of equations. Homogenization represents a promising method to overcome this problem. A two-scale finite element method is proposed to efficiently compute the losses in laminated media with nonlinear material properties. The method based on the magnetic vector potential A is described. In the finite element assembly the laminates are considered individually to account for the nonlinearity. A rather coarse finite element grid suffices to approximate the losses sufficiently accurate. The accuracy and the computational costs of the proposed method are shown by a numerical example.

Index Terms—Eddy currents, finite element methods, laminates, numerical simulation.

I. INTRODUCTION

Brute force methods apply either an anisotropic electric conductivity [1] - [2] or prescribe a current vector potential having a single component normal to the lamination [3] in finite element models. Considering a decomposition of the total magnetic flux into a main magnetic flux parallel to the lamination and a magnetic stray flux perpendicular to the lamination, the solution obtained by the above methods is frequently corrected in a second step exploiting different approaches, for example [4] for 3D problems.

Strictly speaking, the decomposition above is not admissible. This holds particularly for the nonlinear case. Consequently these methods fail. Homogenization methods, where the total magnetic flux is computed in one step, have been proposed using a magnetic scalar potential [5] or a magnetic vector potential [6], respectively. Both methods are able to solve static magnetic fields. Homogenization methods for eddy current problems have been presented for instance in [7].

Previous works [8] - [9] were restricted to linear materials. In the present work a two-scale finite element method (TSFEM) has been developed with the magnetic vector potential A describing eddy currents in laminated iron with a nonlinear magnetization curve. The method is capable to treat a laminated media efficiently as a bulk without the necessity to model the laminates individually. The accuracy and the computational costs of TSFEM are studied by comparing the solution obtained by TSFEM with the reference solution obtained by standard finite element method (FEM).

II. NUMERICAL METHOD

A. Eddy Current Problem

1) *Nonlinear Eddy Current Problem in the Time Domain:* The eddy current problem to be solved is sketched in Fig. 1. It consists of a laminated material Ω_m enclosed by air Ω_0 , i.e., $\Omega = \Omega_m \cup \Omega_0$ with boundary Γ . The material parameters are the nonlinear magnetic permeability μ and the electric conductivity σ . The eddy current problem with the magnetic vector potential A in the time domain reads as

$$\begin{aligned} \text{curl}(\mu^{-1}(A)) \text{curl} A + \sigma \frac{\partial}{\partial t} A &= \mathbf{0} \quad \text{in } \Omega = \Omega_m \cup \Omega_0, \quad (1) \\ A \times \mathbf{n} &= \boldsymbol{\alpha}, \quad \text{on } \Gamma \quad (2) \end{aligned}$$

where t stands for the time.

2) *Variational Formulation:* Equations (1) and (2) yields the following formulation. Find $A_h \in \mathcal{V} := \{A_h \in \mathcal{V}_h : A_h \times \mathbf{n} = \boldsymbol{\alpha}_h \text{ on } \Gamma\}$, such that

$$\int_{\Omega} \mu^{-1}(A_h) \text{curl} A_h \text{curl} \mathbf{v}_h \, d\Omega + \frac{\partial}{\partial t} \int_{\Omega} \sigma A_h \mathbf{v}_h \, d\Omega = \mathbf{0} \quad (3)$$

for all $\mathbf{v}_h \in \mathcal{V}_0 := \{\mathbf{v}_h \in \mathcal{V}_h : \mathbf{v}_h \times \mathbf{n} = \mathbf{0} \text{ on } \Gamma\}$, where \mathcal{V}_h is a finite element subspace of $H(\text{curl}, \Omega)$. Index h indicates finite element discretization.

B. Two-Scale Homogenization

1) *Two-Scale Ansatz:* The two-scale ansatz

$$A = A_0 + \phi(0, A_1)^T + \nabla(\phi w) \quad (4)$$

has been assumed, where A_0 stands for the mean value, A_1 and w are scalar quantities, respectively, and ϕ is the periodic micro-shape function considering the periodic structure of a laminated stack as shown in Fig. 2, where d_1 and d_2 are the thickness' of iron and air layers, respectively.

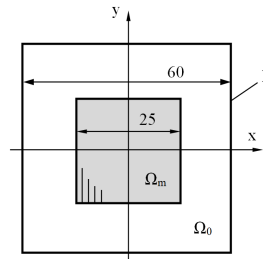


Figure 1: Eddy current problem (dimensions in mm).

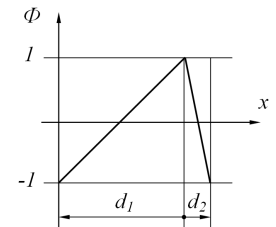


Figure 2: Micro-shape function.

2) *Variational Formulation*: The two-scale ansatz (4) and the variational formulation (3) leads to the variational formulation for the homogenization method: Find $(A_{0h}, A_{1h}, w_h) \in \mathcal{W} := \{(A_{0h}, A_{1h}, w_h) : A_{0h} \in \mathcal{V}_h, A_{1h} \in \mathcal{U}_h, w_h \in \mathcal{W}_h \text{ and } A_{0h} \times \mathbf{n} = \boldsymbol{\alpha}_h \text{ on } \Gamma\}$, such that

$$\int_{\Omega} \mu^{-1}(A_h) \text{curl } A_h \text{curl } v_h d\Omega + \sigma \frac{\partial}{\partial t} \int_{\Omega} \sigma A_h v_h d\Omega = 0 \quad (5)$$

for all $(v_{0h}, v_{1h}, q_h) \in \mathcal{W}_0 := \{(v_{0h}, v_{1h}, q_h) : v_{0h} \in \mathcal{V}_h, v_{1h} \in \mathcal{U}_h, q_h \in \mathcal{W}_h \text{ and } v_{0h} \times \mathbf{n} = \mathbf{0} \text{ on } \Gamma\}$, where \mathcal{V}_h is a finite element subspace of $H(\text{curl}, \Omega)$, \mathcal{U}_h of $L_2(\Omega_m)$ and \mathcal{W}_h of $H^1(\Omega_m)$, respectively, and ϕ is in the space of periodic $H_{per}^1(\Omega_m)$ functions.

C. Numerical Method

1) *Nonlinear Ordinary Differential Equation System*: Carrying out the integrals in (5) yields the matrix equation

$$S(u_h(t))u_h(t) + M \frac{\partial}{\partial t} u_h(t) = f_h(t) \quad (6)$$

with the unknown solution vector $u_h(t)$ with respect to time.

2) *Time Stepping Scheme*: The Backward Euler method were used to solve the nonlinear algebraic system of equations

$$S(u_{h,i+1})u_{h,i+1} + M \frac{u_{h,i+1} - u_{h,i}}{\Delta t} = f_{h,i}, \quad (7)$$

where

$$u_{h,i} = u_h(t_i) \quad (8)$$

at the time instant t_i , Δt means the time step and $u_{h,i}$ and $u_{h,i+1}$, respectively, are the known and unknown solution vector.

3) *Newton's method*: Due to the nonlinearity in the stiffness matrix $S(u_{h,i+1})$, see (7), Newton's method is used:

$$u_{h,i+1}^{(l+1)} = u_{h,i+1}^{(l)} - (\Delta t S'(u_{h,i+1}^{(l)}) + M)^{-1} \cdot (M u_{h,i} - M u_{h,i+1}^{(l)} - \Delta t S(u_{h,i+1}^{(l)}) - \Delta t f_{h,i+1}) \quad (9)$$

The superscript l stands for the l^{th} step in the Newton's method and $S'(u_{h,i+1}^{(l)})$ for the linearization of $S(u)$ in $u_{h,i+1}^{(l)}$. Contrary to the linear case, where the material properties have been averaged across the laminates [9], this is not feasible for the nonlinear case, because the permeability μ depends on the actual solution.

III. NUMERICAL EXAMPLE

Dirichlet boundary conditions were prescribed by $|A_0 \times \mathbf{n}| = 0.004 \text{Vs/m}$ on Γ , see Fig. 1. A thickness of the laminates of 0.25mm , an unfavorable fill factor of 0.9, a conductivity of $\sigma = 2 \cdot 10^6 \text{S/m}$ and a frequency of $f = 50 \text{Hz}$ were selected. The iron stack consists of 100 laminates. The magnetization curve used in the simulations is shown in Fig. 3. Losses obtained once by a finite element model considering the laminates individually (reference solution (RS)) and once by TSFEM are compared in Fig. 4. The agreement is excellent. The required number of degrees of freedom are summarized in Tab. 1. The total number of degrees of freedom can be reduced by a factor of about 31 in this example. Thus, the

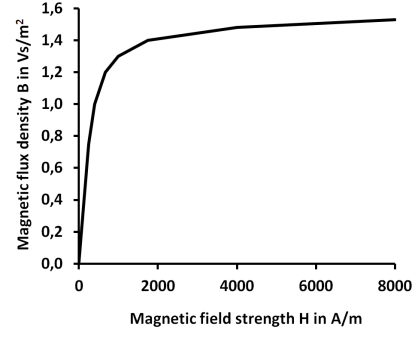


Figure 3: Magnetization curve of an electric sheet.

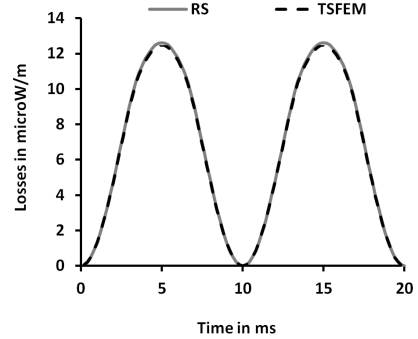


Figure 4: Comparison of eddy current losses.

computational effort of TSFEM is much smaller than that of standard FEM.

Table 1: Number of degrees of freedom.

	Total No.	$H(\text{curl}, \Omega)$	$L_2(\Omega_m)$	$H^1(\Omega_m)$
RS	123564	123564	-	-
TSFEM	4706	3316	692	698

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