Nonlinear evolution of axisymmetric fusion plasmas with three-dimensional volumetric conductors

Fabio Villone, Lucio Barbato, Stefano Mastrostefano, Salvatore Ventre
DIEI, Università di Cassino e del Lazio Meridionale
Via Di Biasio 43, 03434, Cassino (FR), Italy
villone@unicas.it

Abstract—**This paper presents a self-consistent coupling of nonlinear Magneto-Hydro-Dynamic (MHD) equations with eddy currents equations in volumetric three-dimensional conductors, for the analysis of plasma evolution in fusion devices. A differential formulation is used inside the plasma, solved with second-order triangular finite elements; an integral formulation is employed in conductors, resorting to edge elements; a suitable coupling surface is considered in between. Several test cases are presented, showing the effectiveness of the proposed method.**

Index Terms—**Fusion reactors; Tokamaks; Plasma stability.**

I. INTRODUCTION

The main thermonuclear fusion devices, under construction or design, have such high performances to require a special care in the dimensioning of various components. Specifically, the electromagnetic interaction of the plasma with the surrounding conducting structures rules the dimensioning of many crucial components of a fusion device (magnets, vacuum vessel, plasma facing components, etc.).

A particular concern for ITER [1] (a device under construction in Europe, with the contribution of six international partners) are disruptions [2], a sudden loss of magnetic confinement with subsequent release of the magnetic and thermal energy stored in the plasma to surrounding structures. Due to electromagnetic forces and heat loads, the plasma disruptions have a serious impact on the operational lifetime of several components and in extreme cases seriously damage the integrity of fusion devices themselves.

In order to extrapolate the available experimental data to next-generation devices, reliable and complete computational tools are needed. Presently, several modelling approaches are available for the analysis of disruptions, e.g.:

- nonlinear 2D plasma evolution with 2D conductors [3];
- nonlinear 3D plasma evolution, with 2D structures [4];
- plasma evolution in terms of current-driven filaments with 3D conductors [5];
- linearized plasma evolution with 2D/3D conductors [6-8].

Evidently, none of them is fully satisfactory, due to specific limitations and ranges of applicability. Consequently, the analysis of disruptions has been labelled as one of the top priorities in modelling advances by the scientific community.

This paper reports a substantial advance with respect to the state of the art, presenting a computational tool, called CarMa0NL, with the unprecedented capability of simultaneously considering three-dimensional effects of conductors surrounding the plasma and the inherent non-linearity of the plasma behaviour itself.

II. MATHEMATICAL AND NUMERICAL FORMULATION

Calling Ω the region accessible by the plasma and Ω_e the exterior region, the mathematical model is:

$$\begin{gathered} L\,\psi = \mu_0\, j_\varphi(\psi) \text{ in } \Omega \\ \text{Conductors equations in } \Omega_e \end{gathered} \tag{1}$$

where ψ is the magnetic flux per radian, L is the Grad-Shafranov operator and the nonlinear function $j_\varphi(\psi)$ is the toroidal current density in the plasma. On the boundary we can impose $\psi|_{\partial\Omega} = \hat{\psi} = \hat{\psi}_p + \hat{\psi}_e$, where the suffix "*p*" (resp. "*e*") indicates the contribution of the plasma currents inside Ω (resp. external currents). We give a weak form of (1):

$$-\int_\Omega \frac{1}{r}\nabla\psi\cdot\nabla w\, d\Omega + \int_{\partial\Omega}\frac{1}{r}\frac{\partial\psi}{\partial n} w\, dS = \int_\Omega \mu_0 j_\varphi(\psi) w\, d\Omega \tag{2}$$

where w is a suitable test function. Giving a 2D finite elements discretization of Ω, and calling λ_i the hat function related to the i-th node of the mesh, we expand ψ in Ω as:

$$\psi = \sum_{i\in N_i}\psi_i\lambda_i + \sum_{j\in N_b}\hat{\psi}_j\lambda_j \tag{3}$$

where N_i (resp. N_b) is the set of indices of the nodes inside Ω (resp. on $\partial\Omega$). Using the Galerkin method, with suitable definitions [9] (2) becomes:

$$\underline{\underline{A}}\,\underline{\psi} = \underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\hat{\psi}} \tag{4}$$

Similarly, we have $\underline{\hat{\psi}}_p = \underline{\underline{K}}\,\underline{f}(\underline{\psi})$ [8], so that:

$$\underline{\underline{A}}\,\underline{\psi} = \underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\underline{K}}\,\underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\hat{\psi}}_e \tag{5}$$

The flux due to external currents in 3D conductors is computed with the same approach used in the CarMa code [6, 7]. Calling $\underline{I}$ the 3D currents, using an integral formulation in the conducting structures, we have:

$$\underline{\underline{L}}\frac{d\underline{I}}{dt} + \underline{\underline{R}}\,\underline{I} + \underline{\underline{M}}\frac{d\underline{I}_{eq}}{dt} = \underline{\underline{F}}\,\underline{V}, \quad \underline{\psi}_e = \underline{\underline{C}}\,\underline{I}, \quad \underline{I}_{eq} = \underline{\underline{S}}\,\underline{f}(\underline{\psi}) \tag{6}$$

where $\underline{V}$ are the voltages fed to electrodes and $\underline{I}_{eq}$ are equivalent toroidal currents that provide the same poloidal magnetic field as plasma outside Ω [7]. We assume no plasma toroidal flux variation Using an implicit Euler scheme:

$$\left(\underline{\underline{L}} + \Delta t\underline{\underline{R}}\right)\underline{I} + \underline{\underline{M}}\,\underline{I}_{eq} = \Delta t\underline{\underline{F}}\,\underline{V} + \underline{c}, \underline{\psi}_e = \underline{\underline{C}}\,\underline{I}, \underline{I}_{eq} = \underline{\underline{S}}\,\underline{f}(\underline{\psi}) \tag{7}$$

where $\underline{c}$ is a known term depending on the previous time step. Combining (5) and (7), we get, with suitable definitions, the following nonlinear system of equations:

$$\underline{\underline{A}}\,\underline{\psi} + \underline{\underline{H}}_1 \underline{f}(\underline{\psi}) + \underline{\underline{H}}_2 \underline{f}(\underline{\psi}) + \underline{\underline{H}}_3\,\underline{V} + \underline{d} = 0 \tag{8}$$

III. EXAMPLES OF APPLICATION

The CarMa0NL code has been applied to ITER. First of all, a 3D discretization is considered, trying to mimic an axisymmetric description (Fig. 1), so that a benchmark with other available codes is possible. A reference equilibrium is considered, with plasma I_p=15 MA, internal inductance l_i=0.8 and poloidal β_p=0.75 (Fig. 1). Two different events have been considered: an instantaneous plasma current drop of 1MA, and an instantaneous β_p drop of 0.2. Such perturbations trigger a vertical instability; Fig. 1-2 show some snapshots of the plasma configuration and the currents induced in the structures. Fig. 3 shows the comparison with the axisymmetric linear code CREATE_L [9], showing excellent agreement until nonlinear effects come into play (e.g. when the plasma "touches the wall", switching from X-point to limiter configuration at 0.17 s). The nonlinear evolution following the β_p drop has been coupled to more realistic description of the structures - see Figs. 4-5, where also the current density and the plasma configurations are reported at some snapshots.

This paper was supported in part by Italian MIUR under PRIN grant no. 2010SPS9B3.

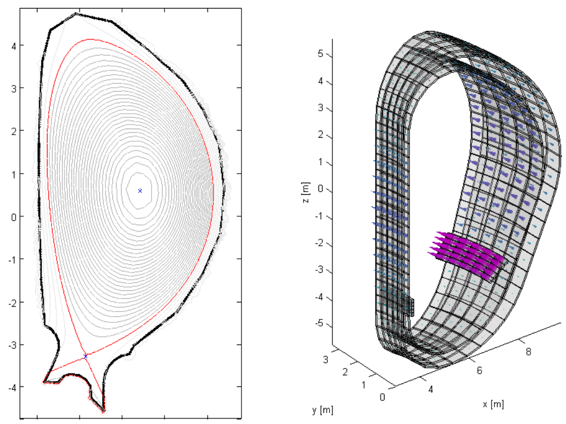

Fig. 1. I_p drop: plasma configuration and conductors currents at 0.07 s

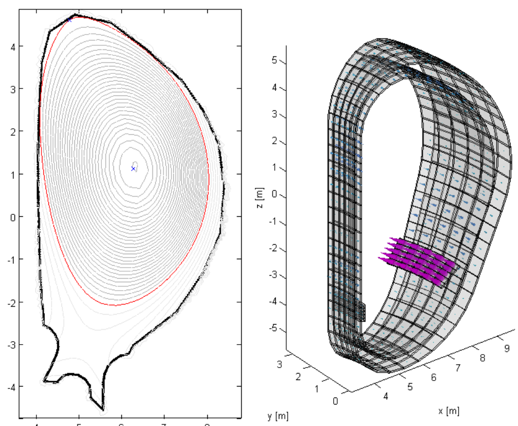

Fig. 2. β_p drop: plasma configuration and conductors currents at 0.03 s

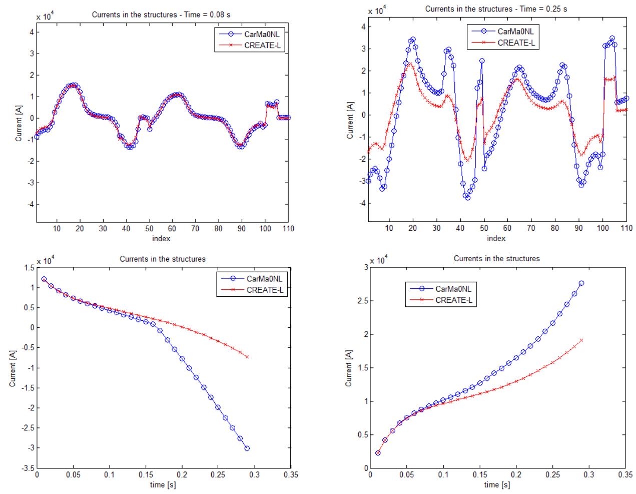

Fig. 3. Comparison of currents in the vessel with the linear code CREATE_L; at t=0.17s the plasma switches from X-point to limiter

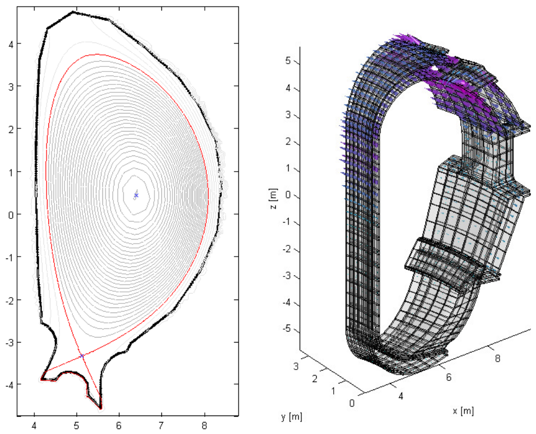

Fig. 5. I_p drop: plasma configuration and conductors currents at 0.25 s

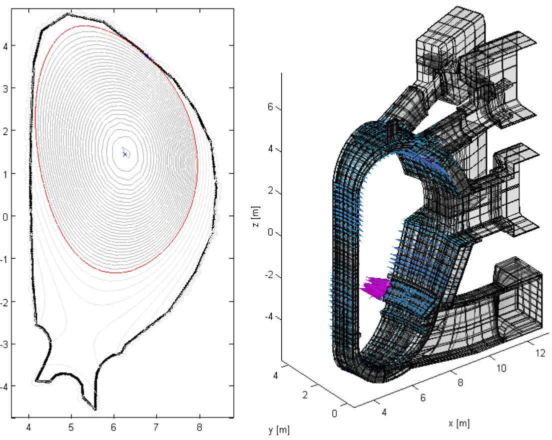

Fig. 5. β_p drop: plasma configuration and conductors currents at 0.30 s

REFERENCES

[1] K. Ikeda, *Nucl. Fusion* **50** (2010) 014002
[2] T. Hender et al, *Nucl. Fusion* **47** (2007) S128
[3] M. Sugihara et al, *Nucl. Fusion* **47** 337-352 (2007)
[4] H.R. Strauss et al., *Physics of Plasmas* **17** 082505 (2010)
[5] R. Albanese et al., *IEEE Trans. Mag.* **46** 2935-2938 (2010)
[6] F. Villone et al, *Nucl. Fusion* **50** 125011 (2010)
[7] F. Villone et al., *Plasma Phys. Control. Fusion* **54** 085003 (2012)
[8] R. Albanese, F. Villone, *Nucl. Fusion* **38** 723 (1998)
[9] F. Villone et al., *Proc. 39th EPS Conference*, paper P5.090 (2012)