Modeling of Magneto-Mechanical Coupling with Magnetic Volume Integral and Mechanical Finite Element Methods

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***Abstract*—The magneto-mechanical coupling is studied using different numerical methods for both physics. The magnetostatic problem is solved with a volume integral method. This method uses the range interactions between magnetizable elements and is particularly well suited to compute energies and magnetic forces, without meshing the air domain. A local application of the virtual work principle adapted to the integral formulations is used to compute the local magnetic forces. These forces are then used in the mechanical formulation solved with finite element method. The same mesh could then be used for magnetostatic and mechanical problems.**

***Index Terms*—magneto-mechanical coupling, integral method, finite element method**

I. Introduction

The modeling of magneto-mechanical devices using magnetic and deformable materials involves the resolution of the magnetostatic and mechanical equations. The first problem is an open boundary problem for the magnetic field whereas the second is restricted to the solid domain. A classical method to solve numerically such problems is the finite element method [1]. However, for devices with a huge volume of free space compared to the active structure or high size ratio between the geometric objects (like MEMS), problems of accuracy could be present [2].

Integral formulations of the magnetostatic field problems are particularly advantageous for the numerical solution of open boundary problems which include magnetic materials since only the active regions containing these materials need to be discretized. So, a volume integral method is used to solve the magnetostatic problem.

To realize the magneto-mechanical coupling, the knowledge of the magnetic forces distribution is necessary. A local application of the virtual work principle adapted to the integral formulations is used. A finite element method is then used to solve the mechanical problem in the solid domain.

This paper presents in the first part the magnetostatic and mechanical formulations. The second part is dedicated to the magneto-mechanical coupling. A numerical example is given.

II. Formulations

A. Magnetostatic problem

Let us consider the following magnetostatic problem. A three dimensional simply connected region Ω_f is filled with a isotropic magnetic material with the known linear magnetic susceptibility χ. Primary sources of magnetic field in which currents flows are associated to the region Ω_j. Both region, Ω_f and Ω_j, are disposed in free space Ω_0 so that these regions do not overlap, $\Omega = \Omega_f \cup \Omega_j \cup \Omega_0$. The linear magnetic behavior law is defined by:

$$\mathbf{M}(\mathbf{r}) = \chi \mathbf{H}(\mathbf{r}), \tag{1}$$

where $\mathbf{M}(\mathbf{r})$ is the magnetization and $\mathbf{H}(\mathbf{r})$ the magnetic field at point of coordinates $\mathbf{r}$. At any point of Ω, the magnetic field is the sum of the reduced magnetic field created by the magnetic material $\mathbf{H}_{red}(\mathbf{r})$ and the magnetic source field created by currents flows $\mathbf{H}_0(\mathbf{r})$.

The simply connected region Ω_f containing no current sources, the following volume integral equation [3] [4] using the total scalar potential Φ is considered:

$$\Phi(\mathbf{r}) + \frac{1}{4\pi} \int_{\Omega_f} \chi \frac{\nabla \Phi(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \, \mathrm{d}\Omega' = \Phi_0(\mathbf{r}), \tag{2}$$

where Φ_0 is the scalar potential whose the magnetic field produced by sources can be derived. A collocation method at mesh nodes is used to solve (2).

B. Mechanical problem

Let us consider the following quasi-static mechanical problem. A three dimensional region Ω_m is filled with isotropic linear elastic material with the known stiffness tensor $[C]$. We note Γ_m the boundary between the region Ω_m and the free space. The behavior law is defined by:

$$\boldsymbol{\sigma} = [C]\boldsymbol{\varepsilon}, \tag{3}$$

where $\boldsymbol{\sigma}$ is the stress tensor and $\boldsymbol{\varepsilon}$ the strain tensor. The local application of the equilibrium equations for a quasi-static mechanical problem leads to the relation:

$$\mathrm{div}\boldsymbol{\sigma} + \mathbf{f} = 0, \tag{4}$$

where $\mathbf{f}$ is the volume force density applied to the mechanical system. According to the small perturbation hypothesis, the strain tensor can be written as:

$$\boldsymbol{\varepsilon}(\mathbf{u}) = \frac{1}{2}(\nabla \mathbf{u} + \nabla^t \mathbf{u}), \tag{5}$$

where $\mathbf{u}$ is the displacement field. Two boundary conditions can be imposed on Γ_m. We note Γ_u and Γ_σ the parts of Γ_m

associated respectively to the conditions on the displacement and the stress with respect to $\Gamma_\sigma \cup \Gamma_u = \Gamma_m$ and $\Gamma_\sigma \cap \Gamma_u = \emptyset$. These conditions are defined by:

$$\mathbf{u} = \mathbf{u}_0 \quad \text{on } \Gamma_u, \tag{6}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{f}^\Gamma \quad \text{on } \Gamma_\sigma, \tag{7}$$

where $\mathbf{n}$ is the outward unit normal vector. The weak formulation associated to the mechanical problem is:

$$\int_{\Omega_m} \boldsymbol{\varepsilon}(\mathbf{u})[\mathbf{C}]\boldsymbol{\varepsilon}(\mathbf{v})\, d\Omega = \int_{\Omega_m} \mathbf{f} \cdot \mathbf{v}\, d\Omega + \int_{\Gamma_m} \mathbf{f}^\Gamma \cdot \mathbf{v}\, d\Gamma \quad \forall \mathbf{v}, \tag{8}$$

where $\mathbf{v}$ is a displacement field verifying the boundary condition (6). A finite element method is used to solve (8).

III. Magneto-mechanical Coupling

This paper considers magneto-mechanical coupling without magnetostriction effect. The force of magnetic origin acting on the solid becomes a source term in (4). The following section presents a method to compute this local magnetic force from an integral formulation.

A. Local magnetic force computation

According to the virtual work principle, the magnetic forces are deduced from the variation of magnetic co-energy, keeping constant the current I during a virtual displacement [5]:

$$\mathbf{F} = \left.\frac{\partial W_{co}}{\partial \mathbf{u}}\right|_{\mathrm{I}}, \tag{9}$$

where I is the electric current, W_{co} the magnetic co-energy, $\mathbf{F}$ the magnetic force and $\mathbf{u}$ a displacement of the system.

Applying this principle at the element level, the nodal magnetic force distribution is obtained by taking the partial derivative of magnetic co-energy with respect to nodal displacements. The nodal magnetic forces $\mathbf{F}_n$ can then be express as [6]:

$$2\mathbf{F}_n = \int_{\frac{\partial \Omega}{\partial \mathbf{u}_n}} \mathbf{M} \cdot \mathbf{B}_0\, d\Omega + \int_\Omega \mathbf{M} \cdot \frac{\partial \mathbf{B}_0}{\partial \mathbf{u}_n}\, d\Omega + \int_\Omega \frac{\partial \mathbf{M}}{\partial \mathbf{u}_n} \cdot \mathbf{B}_0\, d\Omega. \tag{10}$$

The computation of each term of (10) will be presented in the full paper.

B. Application

Let us consider a cantilever beam with a linear and isotropic magnetic behavior (1) and a isotropic linear elastic behavior (3). Two positions for the coil, (a) and (b) (Fig. 1), are considered. The volume integral method (2) is used to solve the magnetostatic problem. The computed magnetic fields are presented in Fig. 2a. The virtual work principle (10) is applied to compute the nodal local magnetic forces which are presented in Fig. 2b. These computed local magnetic forces are validated by an application of the virtual work principle with a finite element approach. The cases (a) and (b) are equivalent to a beam problem with respectively bending and traction stresses. We suppose that the magnetic force is the unique volume force density acting on the beam. After discretization, the corresponding source term in (8) is directly the previous computed nodal magnetic forces. The mechanical problem is then solved and the computed Von Mises stress and displacement fields are presented in Fig. 2c and Fig. 2d.

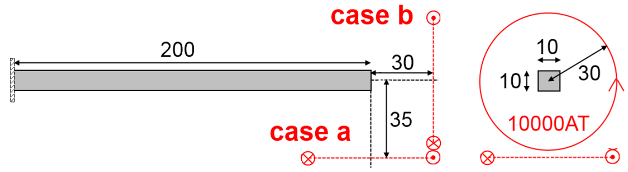
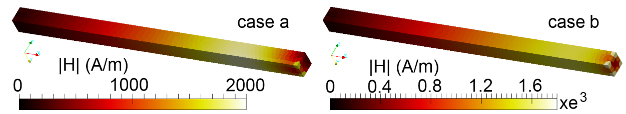
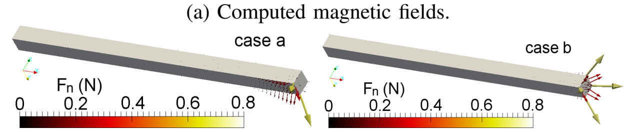


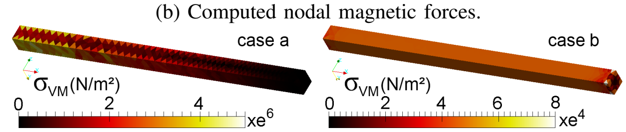
Figure 1: Description of the application.



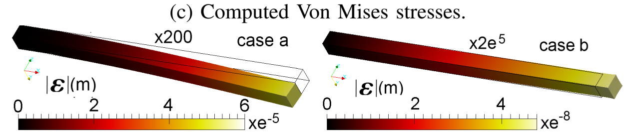
(a) Computed magnetic fields.



(b) Computed nodal magnetic forces.



(c) Computed Von Mises stresses.



(d) Computed displacements (200 and $2e^5$ amplification factors).

Figure 2: Results of the beam application.

C. Conclusion

A magneto-mechanical coupling problem is solved with a volume integral method for the magnetic part and a finite element method for the mechnical part. The volume integral method allows to not mesh the free space and is well suited to compute the energies. These last are used thanks to the virtual work principle to compute the magnetic force density which becomes a source term in the mechanical formulation. The same mesh could be used for magnetostatic and mechanical problems. A futur work can be the investigation of the nonlinear case including a material magneto-mechanical coupling.

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