Iron loss calculation in steel laminations at high frequencies

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***Abstract*—The strong interaction between hysteresis and eddy currents in electrical steel laminations cannot be resolved without a strongly coupled finite element modelization. This paper presents such a model, together with a systematic methodology to identify the free parameters from standard measurements (Epstein frame or Single Sheet Tester). As the model is based on a thermodynamic analysis, the identified parameters are true material parameters, that characterize the material irrespective of load and geometry. They can therefore be used to quantitatively evaluate fields and losses at higher frequencies and in the presence of higher harmonics.**

***Index Terms*—Hysteresis modeling, eddy-currents, homogenization, vector-hysteresis, higher harmonics, iron losses.**

I. Introduction

Induced currents appear at various geometrical scales in ferromagnetic samples subjected to a time-varying magnetic field $\mathbf{h}(t)$. Following Bertotti [1], two main mechanisms can be distinguished. On the one hand, induced currents that result directly from the variation of the external magnetic field and loop up over pathes of macroscopic dimension are called eddy currents, or Foucault currents. They depend on the geometry of the sample, and their rate of variation is directly determined by that of the applied field, $\partial_t \mathbf{h}(t)$. On the other hand, a microscopic induction mechanism also exists associated with the broken (jerky) motion of Bloch walls (Barkhausen effect) as the magnetic polarization changes in the sample. The dynamics of this motion, which is ruled by the microstructure, determines the intensity and the distribution of the microscopic induced currents, whose associated Joule losses are conventionnally called hysteresis losses. The density of hysteresis losses does not depend on the geometry of the sample, neither on the rate of variation of the applied magnetic field (hysteresis is a local and quasi-static phenomenon), but it depends, at each point in the sample, on the local maxima attained by the field $\mathbf{h}(t)$ all through the magnetisation history. The term iron losses generically covers the losses associated with both phenomena.

The difficulty in modelling iron losses is associated with the fact that ferromagnetic cores generally come up as stacks of thin isolated electrical steel laminations (whose typical thickness is between 0.2 mm and 1.0 mm). Homogenization techniques have been proposed to model such composite magnetic structure [2], [3]. They however assume simplified forms for the current density across the lamination (resulting e.g. from the solution of the eddy current problem) and disregard hysteresis in general. We believe that applying homogenization techniques is premature because a reliable mesoscopic model fails. The interaction between hysteresis and eddy currents is indeed so strong that it cannot be resolved without an explicit field modelization inside the laminations. The aim of this paper is to present such a strongly coupled model that addresses eddy currents (including skin effect) and hysteresis simultaneously. The proposed parametrized model is based on a thermodynamic analysis, and a methodology to identify the material parameters from standard Epstein Frame (EF) or Single Sheet Tester (SST) experiments is described. Once identified, the material parameters can be used to analyze the behaviour of laminated cores at higher frequencies and in the presence of higher harmonics.

II. Hysteresis material model

The hysteresis model follows from the expression of the conservation of energy in the material $\dot{\Psi} = \mathbf{h} \cdot \dot{\mathbf{b}} - D$ with Ψ the internal energy density, $\mathbf{h} \cdot \dot{\mathbf{b}}$ the rate of magnetic work and D a dissipation function. In order to appropriately account for the susceptibility of empty space, the induction field, $\mathbf{b} = \mathbf{J}_0 + \mathbf{J}$, is written as the sum of an empty space magnetic polarization and a material magnetic polarization associated with the presence of microscopic moments attached to the atoms of the ferromagnetic sample. The energy density is

$$\Psi(\mathbf{J}_0, \mathbf{J}) = \frac{\mathbf{J}_0^2}{2\mu_0} + u(\mathbf{J}) \tag{1}$$

with μ_0 is the magnetic permeability of vacuum, and its time derivative writes

$$\dot{\Psi} = \frac{\mathbf{J}_0}{\mu_0}\dot{\mathbf{J}}_0 + \mathbf{h}_r \cdot \dot{\mathbf{J}} \quad \text{with} \quad \mathbf{h}_r := \partial_{\mathbf{J}} u. \tag{2}$$

The dissipation function

$$D = \kappa|\dot{\mathbf{J}}| = \mathbf{h}_i \cdot \dot{\mathbf{J}} \quad \text{with} \quad \mathbf{h}_i := \partial_{\dot{\mathbf{J}}} D = \kappa \frac{\dot{\mathbf{J}}}{|\dot{\mathbf{J}}|} \tag{3}$$

describes hysteresis as the magnetic analogous of a dry friction force, whose physical origin is the pinning effect that opposes the motion of Bloch walls. Conservation of energy now yields

$$(\mathbf{h} - \mathbf{J}_0/\mu_0) \cdot \dot{\mathbf{J}}_0 + (\mathbf{h} - \mathbf{h}_r - \mathbf{h}_i) \cdot \dot{\mathbf{J}} = 0 \qquad \forall \dot{\mathbf{J}}_0, \dot{\mathbf{J}}. \tag{4}$$

As the state variables $\mathbf{J}$ and $\mathbf{J}_0$ are arbitrary, the factors between parenthesis must vanish, and the constitutive relationships of the material are obtained, namely $\mathbf{J}_0 = \mu_0 \mathbf{h}$ and

$$\mathbf{h} - \mathbf{h}_r - \mathbf{h}_i = 0 \quad \Rightarrow \quad \mathbf{h} - \partial_{\mathbf{J}} u - \kappa \frac{\dot{\mathbf{J}}}{|\dot{\mathbf{J}}|} = 0. \tag{5}$$

In reality, the pinning strength κ is not a constant but obeys a statistical distribution, which can be represented with controllable accuracy by combining several models like (5) [4], [5].

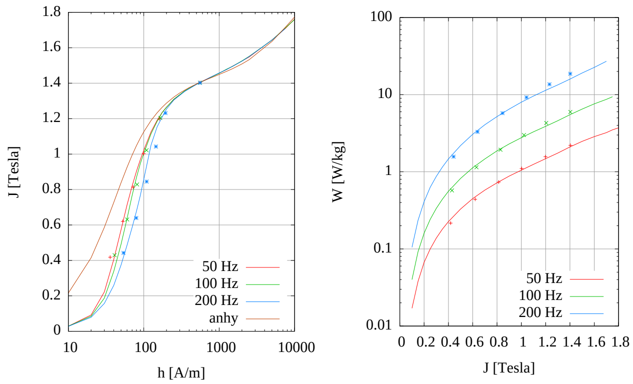

Figure 1: Comparison between measured data for M25050A FeSi non-grainoriented electrical steel at 50Hz, 100Hz and 200Hz (solid lines) and calculated data (points): Virgin and anhysteretic curve (left), iron losses (right).

III. Finite element model

The measured quantities in EF or SST experiments are currents and fluxes related to the magnetic field $\mathbf{h}$ and the average induction $\langle \mathbf{b} \rangle$ across the lamination thickness. Since by construction those quantities are uniform across measured samples, it is sufficient for our purposes to work with a finite element (FE) model that consists of a 1D formulation of the eddy current problem :

$$\int_{\Omega} \left(\partial_t \mathbf{b} \cdot \mathbf{h}' + \sigma^{-1} \mathrm{curl}\, \mathbf{h} \cdot \mathrm{curl}\, \mathbf{h}' \right) d\Omega = 0 \quad \forall \mathbf{h}' \tag{6}$$

with $\mathbf{h} = (0, h(z), 0)$. A h-field formulation is chosen because the magnetic field is the natural driving quantity for the irreversible constitutive relationship (5). Considering a lamination of thickness $2d$ with an upper surface normal vector $\mathbf{n} = (0, 0, 1)$, the domain of analysis Ω is a line parallel to $\mathbf{n}$, across half the thickness, and far from the edges. The boundary condition at the center of the lamination is $\mathrm{curl}\, \mathbf{h}(0) \times \mathbf{n} = 0$, whereas a given external field $\mathbf{h}(d)$ is applied at the surface of the lamination. Iron losses per unit surface are given by the flux of the Poynting vector $\sigma^{-1} \mathrm{curl}\, \mathbf{h}(d) \times \mathbf{h}(d)$ across the lamination surface. The details of the implementation will be given in the full paper.

IV. Parameter identification

Standard electric steel lamination measurements are obtained under sinusoidal $\langle \mathbf{b} \rangle$-field conditions. In order to identify them with FE simulations, the applied $\mathbf{h}$-field that yields a sinusoidal flux through the lamination must be determined. This can be done iteratively (See full paper). Fig. 1 shows the match obtained with a minimal number of material parameters. A very good match over a quite large range of field intensities (up to 1.4 Tesla) and frequencies (up to 200Hz) is observed. It is remarkable that the large amount of measured data can be quantitatively reproduced with so few parameters (7 in this case). This indicates that the physical model based on the dry friction analogy is close enough to the reality.

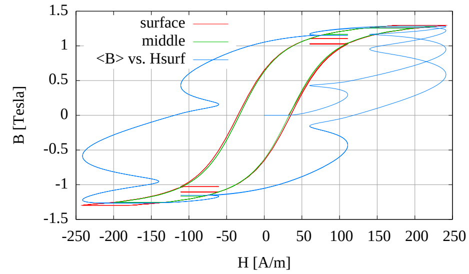

Figure 2: Comparison between true $(\mathbf{h}, \mathbf{b})$ curves, at the surface and at the center of the lamination, and the measured loop $(\mathbf{h}, \langle \mathbf{b} \rangle)$.

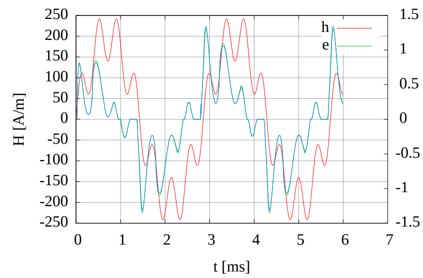

Figure 3: Time evolution of $\mathbf{h}$ and $\mathbf{e}$ at the surface of the lamination, the product of which is the delivered power.

V. Application

The identified parameters fully characterize the material, irrespective of load and geometry. They can be used in 2D or 3D models, or to calculate iron losses under loads for which measurements are hard to obtain or unavailable, in particular at higher frequencies and in the presence of higher harmonics. Figures 2 and 3 illustrate the rather complex phenomenology in such situations. A 350Hz magnetic field of magnitude 200A/m superimposed with a 7^{th} higher harmonic (2450Hz) of magnitude 60A/m has been applied to the lamination model. The true and the measured hysteresis loops are compared in Fig. 2. Figure 3 shows the complex shape of the magnetic and electric fields, whose product $\mathbf{h} \times \mathbf{e}$, the Poynting vector, gives the iron losses. This model can be combined with a homogenization approach like the one proposed in [**?**] to address macroscopic devices such as, e.g., electrical machines.

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