A Multiobjective Approach of Differential Evolution Optimization Applied to Electromagnetic Problems

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Abstract — Differential Evolution (DE) algorithms belongs to a class of evolutionary optimization techniques that uses stochastic approach to solve the optimization problems. It was proposed by Storn and Price in its mono-objective form, and since then it has been reaching great results, like global convergence faster than other classical evolutionary algorithms. This paper proposes a multiobjective approach, combining the evolutionary mechanism from DE with some ideas extracted from the Strength Pareto Evolutionary Algorithms (SPEA), proposed by Ziztler and Theile. This approach was applied to electromagnetic problems, and results are shown for demonstrate its applicability and robustness.

***Index Terms*— Optimization, Electric Machines**

I. Introduction

Frequently, during the electromagnetic device design, the engineer has to use optimization tools to find a good solution for problems with conflicting objectives. Just the process of selecting an optimization algorithm that fits well on the problem is very complex, since it is correlated with the engineer's previous experience with the electromagnetic device design and the optimization methods. Recently literature contains established stochastic optimization methods for solving electromagnetic problems in a mono [1] or multiobjective form [2].

This paper presents a proposal of new algorithm to solve multiple objective problems (in a fast, simple and robust way) by combining the evolutionary mechanism from Differential Evolution (DE) [3] and the external Pareto archiving, as proposed in [4].

The contribution of the current work is to present the theoretical approach of this method, demonstrating part of its potential by optimizing the design of a Brushless DC motor. The problem has two objectives, five-degrees of freedom and some constrains.

II. The proposed Algorithm

The multiobjective approach of Differential Evolution Algorithm (*MultiDE*) was designed to be as simple as the original (mono-objective) version. In fact, the evolutionary mechanism from DE is mainly used to create each "test population" and explore the space of variables. In each turn of the optimization process a Pareto Front is created and compared with another one, which is archived on external file [4] that is frequently updated until the run stops.

Before run the optimization, some parameters should be set: the Mutation Factor - MF (real number between 0 and 0.5); the Crossover Rate - CR (real number between 0 and 1); the population size - NP and the maximum size of the Pareto Front.

When started, a first population is randomly created and analyzed, eliminating non-feasible solutions. The other solutions, are ranked according to their strength, i.e., and the procedure proposed on [4] is performed. After that, the selection step is firstly called.

The *multiDE* is established as follows: three elements are randomly chosen, one from the external Pareto (Z_{par}) and other two from the previous population ($X_{i_2}(t)$, and $X_{i_3}(t)$). A difference between the last two elements is weighted by MF and added to the first element from Pareto, resulting on new mutated vector ($X_{i_1}(t+1)$), as showed at the equation (1), where i = 1 … NP is the individual's index of population and t is the current generation:

$$X_{i_1}(t+1) = Z_{par} + MF.\left(X_{i_2}(t) - X_{i_3}(t)\right) \qquad (1)$$

Then, in order to increase diversity, the mutated vector is passed by a "Crossover" (2). On this step, $X_{i_1}(t+1)$ is mixed with Z_{par} as follows:

$$X_{i_{1,j}}(t+1) = \begin{cases} X_{i_{1,j}}(t+1) & if \quad (randb(j) \leq CR) \\ Z_{par,j} & if \quad (randb(j) > CR) \end{cases} \qquad (2)$$

In (2), *randb(j)* is a real random number between 0 and 1, being compared with CR for every j^{th} position of vector of variables. If *randb(j)* is lower than CR, the j^{th} mutated position is maintained, otherwise the j^{th} Pareto Element will take this position.

After that, those new vectors of the current generation are compared with the external Pareto Set. All solutions are globally ranked by using the strength Pareto concept, as proposed in [4] and a selection is done, surviving on the external archive only the non-dominated ones. If the Pareto-set number of elements is greater than a prescribed maximum, a cluster reduction should be performed, as proposed in SPEA.

The process keeps running until at least one of the stop criteria is reached. Those criteria can be the number of evaluations, deviation between runs, etc. Usually, to obtain more accurate results, the optimization process can be run several times, and then, all different solutions may suffer a Pareto reduction reaching the final response.

The advantage on this method is its simplicity, low number of control parameters and computational requirements. Its code can be easily modified and updated. To validate the proposed algorithm, tests were exhaustively done to determine the behavior of this method on different kind of problems and parameter sets.

III. Brushless DC Wheel Motor Problem

This problem is a benchmark proposed in [5] and [6] and one solution of this multiobjective problem is described in [7]. All physical modeling are well described on [5] and [6], so it will be omitted here.

The challenge is to minimize the motor mass and simultaneously maximize its efficiency, finding the value of five input variables: the stator diameter (D_s), the magnetic induction in the air gap (B_e), the current density in the conductors (δ), the magnetic induction in the teeth (B_d) and the magnetic induction in the stator back iron (B_{cs}).

A feasible solution should respect constraints for the outer diameter (D_{ext}); the inner diameter (D_{int}); the current on the magnets (I_{max}); the temperature of the magnets (T_a) and the determinant $Det(D_s, \delta, B_d, B_s)$ used for the calculation of the slot height must be positive. Those parameters and constraints are shown at Table I:

TABLE I Parameters And Constraints

		Lower Bound	Upper Bound			
PARAMETERS	Ds [mm]	150	300	**CONSTRAINTS**	Dext	<340 mm
	Be [T]	0.50	0.76		Dint	>76mm
	δ[A/mm²]	2.0	5.0		Imax	≥ 120ºC
	Bd [T]	0.9	1.8		Ta	<120ºC
	Bcs [T]	100	100		Det(Ds,δ,Bd,Bs)	>0

IV. Optimization Results

For the optimization of this problem, 10 independent runs, with 100 individual in each population (NP) and 100 elements on the Pareto Set. The Mutation Factor was set as 0.015 and Crossover Rate as 0.9. The run stops when reach a maximum number of evaluation (2000). It was observed that a good Mutation Factor value for this class of problem should be very low, being also a good choice for some functions of the suite of benchmark multiobjective problems proposed in [8]. The Final Pareto obtained is shown in Fig. 1 and presents good diversity. To observe more accurately the result of optimization, each solution from Final Pareto set was analyzed with respective frequency, is shown at Fig. 2.

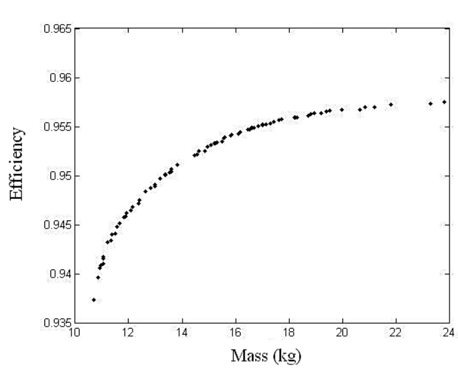

Fig. 1. The Final Pareto Set

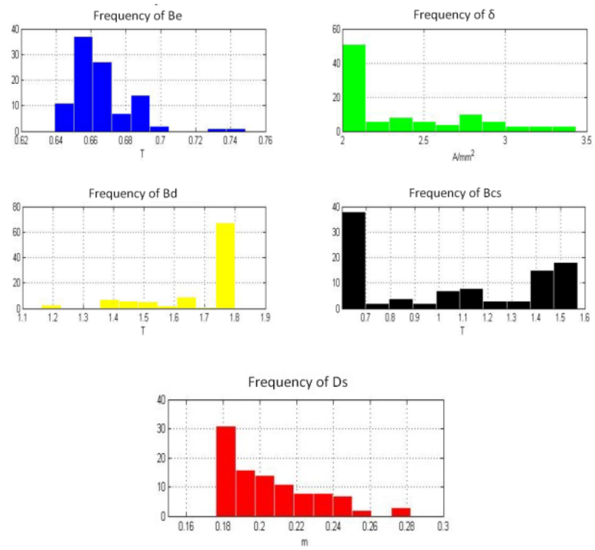

Fig. 2 Frequency of parameters after optimization

It's possible to observe a trend of the physics characteristics of this device, which converge to typical values of this kind of electrical machine design, for example: quite all the non-dominated solutions presents high value (the upper value) for the magnetic induction in the teeth (B_d), and there is a typical value for the induction on the air gap (B_e) close to 0.66 T.

V. Conclusions

In this paper a new multi objective technique for the global optimization of electromagnetic devices was introduced, inheriting some characteristics from its inspiration parents. This proposal of a variant of Differential Evolution algorithm for multiobjective problems attained good results, being another powerful tool to solve this kind of problem. It may be an alternative or a complement to the established methods in many applications.

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