Model Order Reduction of Non Linear Magneto-static Problems based on POD and DEI Methods

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Abstract—**Model Order Reduction Method like Proper Orthogonal Decomposition (POD) can be very efficient in the linear case but meet limitations in the non linear case. In this communication, the Discret Empirical Interpolation Method coupled with the POD method is presented. It is an interesting alternative to reduce large-scale system arising from the discretization of non linear magnetostatic problem coupled with electric circuit.**

Index Terms—**Reduction Method, POD, DEIM.**

I. INTRODUCTION

To describe the behavior of electrical machines coupled with an external electric circuit, the Finite Element Method associated with a time-stepping scheme is often used to solve numerically Maxwell's equations coupled with the circuit equations. When a fine mesh and a small time step are used, the computation time of the large-scale system obtained from the discretization of the Non Linear Partial Differential Equations (NL-PDE) can be prohibitive. To tackle this issue, an alternative is to use model order reduction methods. In the literature, the Proper Orthogonal Decomposition (POD) has been widely used to solve a lot of problems in engineering [1]. This method consists in performing a projection onto a reduced basis of the space domain, the size of the system of equations to solve can be highly reduced. The Snapshot approach is often used to determine the discrete projection operator between the original basis (generating from the mesh) and reduced basis [2]. In computational electromagnetics, the POD method has been applied, for example, to study the behavior of a transformer with a non linear core [3] or to solve an electroquasistatic field problem [4]. In the case of a system with linear PDE, the POD approach can lead to a dramatic reduction of the computation time. In the non linear case, this method is not so efficient anymore due to the computation cost of the non linear term in the reduced system which requires the matrix assembling of the initial problem. To solve this problem, the Discret Empirical Interpolation Method (DEIM) method can be used with the POD approach [5]. This method consists in interpolating the non linear behaviour of the magnetic field on the whole studied domain from evaluations of the non linear behaviour law on a small number of degrees of freedom. The computation time of the non linear term when applying the POD is then highly reduced.

In this communication, the DEIM-POD approach is applied to solve a non linear magnetostatic problem coupled with an electric circuit using the vector potential formulation. First, the numerical model obtained from the formulation is presented. Secondly, the POD with Snapshot method and the

DEIM are developed. Finally, an example is treated where the different non linear models obtained from the POD method and from the coupling of the POD method and DEIM are compared. The results obtained with the reduced model are also compared in terms of accuracy and time calculation with the original Finite Element Model.

II. NON LINEAR MAGNETO-STATIC PROBLEM COUPLED WITH ELECTRIC CIRCUIT

A magnetostatic problem can be solved using the vector potential $\mathbf{A}$ such that $\mathbf{B}(\mathbf{x},t)=\mathbf{curl}\mathbf{A}(\mathbf{x},t)$. To take into account the non linear behaviour of the ferromagnetic material, the fixed point technic can be used. In that case, the magnetic field $\mathbf{H}(\mathbf{x},t)$ can be expressed by $\mathbf{H}(\mathbf{x},t)=\nu_{fp}\mathbf{B}(\mathbf{x},t)+\mathbf{H}_{fp}(\mathbf{B}(\mathbf{x},t))$ with ν_{fp} a constant and $\mathbf{H}_{fp}(\mathbf{B}(\mathbf{x},t))$ a virtual magnetisation. Then, the equations to solve are

$$\mathbf{curl}(\nu_{fp}\mathbf{curl}\mathbf{A}(\mathbf{x},t)) - \mathbf{N}(\mathbf{x})i(t) = \text{-}\mathbf{curl}(\mathbf{H}_{fp}(\mathbf{A}(\mathbf{x},t))) , \quad (1)$$

$$\frac{d}{dt}\int_{D} \mathbf{A}(\mathbf{x},t).\mathbf{N}(\mathbf{x})dD + Ri(t) = v(t) \quad (2)$$

with $\mathbf{N}$, $v(t)$, $i(t)$ and R the unit current density vector, the voltage, the current and the resistor associated with a stranded inductor. The equation (2) enables to take into account the coupling with the external circuit. Using a spatial semi-discretisation of $\mathbf{A}(\mathbf{x},t)$ (in 2D, with the standard nodal elements), the system of ordinary differential equations to solve can be written:

$$\mathbf{M}\mathbf{X}(t) + \mathbf{K}\frac{d\mathbf{X}(t)}{dt} = \mathbf{F}(t) - \mathbf{M}_{fp}(\mathbf{X}(t)) \quad (3)$$

with $\mathbf{X}(t)$ the vector of unknowns of size N_{un} such that $(X_i(t))_{1\leq i \leq Ne} =(A_i(t))_{1\leq i \leq Ne}$ with $A_i(t)$ the value of $\mathbf{A}$ at the node i and $X_{Nun}(t) = i(t)$. $\mathbf{M}$ and $\mathbf{K}$ are matrices and $\mathbf{F}(t)$ and $\mathbf{M}_{fp}(\mathbf{X}(t))$ vectors.

III. MODEL ORDER REDUCTION WITH DEIM-POD

A. Proper Orthogonal Decomposition

In order to reduce the computation time required to solve the previous matrix system, the POD approach can be used [1]. The vector $\mathbf{X}(t)$ is approximated in a reduced basis by a vector $\mathbf{X}_r(t)$ of size N_s ($N_s \ll N_{um}$). To determine a discrete projection operator $\mathbf{\Psi}$ such that $\mathbf{X}(t) = \mathbf{\Psi}\mathbf{X}_r(t)$, the Snapshot approach is generally applied [2]. The system (3) is solved for the solutions (called Snapshots) calculated at the first N_s time steps. The Snapshot matrix $\mathbf{M}_s$ is defined by $\mathbf{M}_s=(\mathbf{X}^j)_{1\leq j\leq Ns}$.

with $\mathbf{X}^j$ the solution $\mathbf{X}(t)$ at the jth time step. A way to express $\mathbf{\Psi}$ is to use the Singular Decomposition Value of $\mathbf{M}_s$,

$$\mathbf{M}_s = \mathbf{V\Sigma W}^t \qquad (4)$$

with $\mathbf{V}_{Nun \times Nun}$ and $\mathbf{W}_{Ns \times Ns}$ orthogonal matrices of eigenvectors and $\mathbf{\Sigma}_{Nun \times Ns}$ the diagonal matrix of the eigenvalues values. The operator $\mathbf{\Psi}$ is equal to $\mathbf{V\Sigma}$. The reduced system to solve can be deduced from (3) such as

$$\mathbf{M}_r \mathbf{X}_r(t) + \mathbf{K}_r \frac{d\mathbf{X}_r(t)}{dt} = \mathbf{\psi}^t \mathbf{F}(t) - \mathbf{\psi}^t \mathbf{M}_{fp}(\mathbf{\Psi X}_r(t)) \qquad (5)$$

$$\text{with } \mathbf{M}_r = \mathbf{\Psi^t M \Psi} \quad \text{and} \quad \mathbf{N}_r = \mathbf{\Psi^t N \Psi}.$$

B. DEIM with POD

In the non linear case, the computational complexity of the vector $\mathbf{f}_{nl}(t) = \mathbf{M}_{fp}(\mathbf{\Psi X}_r(t))$ can be important (see (5)). In fact, it is necessary to evaluate the solution $\mathbf{X}(t)$ in the original basis by $\mathbf{\Psi X}_r(t)$ and to determine the vector $\mathbf{M}_{fp}(\mathbf{X}(t))$. To tackle this issue, an alternative is to apply the DEIM [5]. This approach proposes to approximate the nonlinear function $\mathbf{f}_{nl}(t)$ by combining projections with interpolations. All terms of the vector $\mathbf{f}_{nl}(t)$ don't need to be evaluated anymore but only a small number of them. In 2D, the nodes, where the non linear function is evaluated, are called the DEIM points. We seek for approximating $\mathbf{f}_{nl}(t)$ by the vector $\mathbf{Uc}(t)$ with $\mathbf{U}$ a orthogonal matrix defined by a Snapshot-POD approach in the same way as $\mathbf{\Psi}$ and $\mathbf{c}(t)$ the interpolation function vector. To determine the coefficients of $\mathbf{c}(t)$, N_{deim} distinct rows from the over-determined system are selected by applying a matrix $\mathbf{P}$ such as $\mathbf{P^t f}_{nl}(t) = \mathbf{P^t Uc}(t)$. The algorithm presented in [5] is used to determine the matrix $\mathbf{P} = (\mathbf{I}_i)_{1 \le i \le Ndeim}$ with $\mathbf{I}_i$ the i-th column of the identity matrix $\mathbf{I}_{Nun \times Nun}$. $\mathbf{f}_{nl}(t)$ can be approximated by

$$\mathbf{f}_{nl}(t) \approx \mathbf{U}(\mathbf{P^t U})^{-1} \mathbf{M}_{fp}(\mathbf{P^t \Psi^t X}_r(t)). \qquad (6)$$

In this expression, the calculation of the term $\mathbf{M}_{fp}(\mathbf{P^t \Psi X}_r(t))$ requires just the evaluation at the N_{deim} nodes of the non linear function and not at all nodes as in (5).

IV. APPLICATION

A 2D magnetostatic example, made of a single phase transformer supplied with a 50 Hz sinusoidal voltage at no load, is studied. The non linear magnetic behavior of the core is considered. The 2D spatial mesh is made of 1330 nodes and 2560 triangles. The Euler scheme is used to solve (3) and the time step is fixed to 1ms. Figure 2 presents the evolution of the current obtained from the original model (reference) and the POD and DEIM-POD approaches with 6 snapshots. The number of DEIM points is equal to the number of snapshots. We can observe that the POD model gives results closer to the reference than the DEIM-POD method. Nevertheless, in term of computation time, the DEIM-POD model is faster due to the evaluation of $\mathbf{M}_{fp}(\mathbf{\Psi X}_r(t))$ in (5). With the POD approach, the time computation is close to the one of the reference model. Table I presents the maximal error of the current obtained from the two reduced models and the ratio of the computation

times with the reference model versus the number of snapshots. The DEIM-POD approach requires more snapshots to obtain a result very close to this one of the reference than the POD model. Nevertheless, as the computation time is dramatically reduced, the DEIM-POD approach appears as a very interesting alternative. Figure 3 presents the DEIM points for 10 snapshots. As expected, these points determined automatically by the DEIM are located in saturated area.
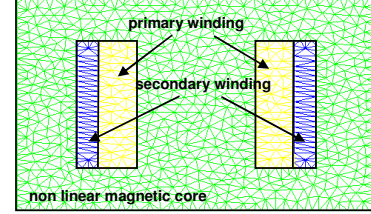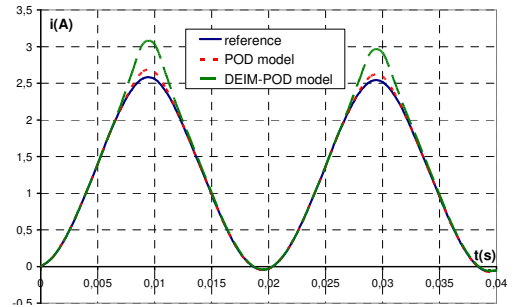

Fig. 1. Example of application

Fig. 2. Evolution of the current

TABLE I
COMPARISON BETWEEN POD AND DEIM-POD MODELS VERSUS THE NUMBER OF SNAPSHOTS

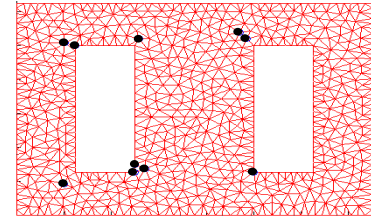| Number of Snapshots	4	8	10
Max error of current (%)– POD model	10.87	0.036	0.026
Max error of current (%) – DEIM-POD model	14.48	2.6	0.06
Time ratio (%)	0.6	2.18	3.67

Fig. 3. DEIM points on the mesh

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