# Fast Frequency-Domain Modeling of Return Stroke including Influence of Lossy Ground

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***Abstract*— A constitutive integral equation (CIE) formulation is proposed to model the lightning return stroke. Using moment of method (MOM), the electromagnetic field radiated by the return stroke considering a lossy ground is determined. To implement a fast numerical and rigorous approach, firstly, an exact image theory is employed to accelerate the computation of Sommerfeld integral; and, secondly, a translational mesh is constructed under the thin-wire approximation. Such grid representation and the convolution properties of the integral kernel equip the fast Fourier transform (FFT) to efficiently solve the MOM matrix. The comparison of the numerical results on a case study with existing one positively confirms the proposed method.**

***Index Terms*—Moment method, return stroke, fast Fourier transform (FFT), exact image.**

## I. Introduction

The evaluation of electromagnetic field generated by lightning strokes is of extreme importance in the lightning protection. In engineering applications, the experiments have evidenced that, among all the stages of a lightning flash, the first return stroke causes most catastrophes, and several simplified models are introduced to qualitatively simulate this process to some extent [1]-[2]. Most of these models postulate a vertical lightning channel model without any branch and a certain source at the channel base. For example, the engineering return-stroke models directly assume a spatial and temporal current distribution of the lightning channel based on the base current while the transmission line models consider the lightning channel as a distributed circuit. Clearly, both of these models produce an incorrect electromagnetic field distribution in the vicinity of the lightning channel. Based on the Maxwell's equations, the electromagnetic return stroke should be stimulated using a more rigorous full-wave solution technique, which is very suitable for the numerical analysis of electromagnetic interactions of different systems in a wide range of distances.

Another issue in lightning stroke studies is to model the ground conductivity [3]. In this aspect, the ground has been assumed to be a perfect conducting plane in order to obtain a closed-form expression in many available studies. Obviously, this assumption is reasonable for very close distance range, and high conductivity ground. However, for most of distance ranges, the horizontal component of the electric field radiated by the return stroke is significantly affected by the ground conductivity. Based on this argument, several approaches were proposed to modify the results obtained from the perfect conducting assumption. Among these approaches, the Cooray-Rubinstein approach permits to provide relatively satisfactory results for all considered ranges. However, it is observed that, at some meters above the ground, the horizontal intensity of the field is much smaller than that of the vertical component, and its propagation effects, such as rise time and attenuation of higher frequency components, are changed.

A frequency-domain electromagnetic model, as firstly proposed by Sommerfeld, is the most rigorous and promising candidate in studying lightning electromagnetics taking lossy ground effect into account. However, the numerical implementation of such model suffers from large electrical scale and expensive frequency sweep required to compute transient process. In this point of view, the finite difference time domain (FDTD) method is commonly used to match the characteristics of lightning stroke being a typical transient electromagnetic problem having electrically large geometry. However, as the frequency-dependent ground conductivity, open boundary are required to accurately model the lightning stroke, and it will become awkward to implement.

In this paper, several proposals are proposed to provide a fast moment method for the computation of Sommerfeld approach. More specifically, the Sommerfeld integral in the Green's function is evaluated by using an exact image theory, and the solution of MOM equations is accelerated by using fast Fourier transform (FFT). To verify the effectiveness of the proposed method, a numerical example is reported.

## II. Formulation of the Electromagnetic Model of the Return Stroke

The proposed electromagnetic model is shown in Fig. 1. In this model, a vertical lightning channel is used, and its medium parameters are denoted as $\sigma_2$, $\varepsilon_2$, which can be used to estimate the propagation constant of the current. To take into account of the influence of the finite ground in the calculation of the radiated field, the infinite ground is assumed with a conductivity $\sigma_1$ and permittivity $\varepsilon_1$.

The total electric field is the sum of the incident field $\mathbf{E}_i$ and the scattering field $\mathbf{E}_s$, i.e.,

$$\mathbf{E} = \mathbf{E}_s + \mathbf{E}_i \tag{1}$$

The return stroke, which is presented as a body made of an inhomogeneous medium, is surrounded by airs. Given the Green's function of the half-space problem, the scattering electric field $\mathbf{E}_s$ can be represented as

$$\mathbf{E}_s = \int_\Omega \mathbf{G} \cdot \mathbf{J} dv \tag{2}$$

where, Ω denotes the space of the return stroke, **J** is the unknown current in the body.

The constitutive relations are known to be of the form

$$\mathbf{J} = [\sigma + j\omega(\varepsilon_r - 1)\varepsilon_0]\mathbf{E} \tag{3}$$

where $\varepsilon_r$ is the relative permittivity of the medium.

By substituting (2) and (3) into (1), it is easy to give the constitutive integral equation [4]. If the thin-wire approximation is applied, only the $z$ component, as shown in Fig. 1, is hold as follows,

$$\frac{J_z}{\sigma + j\omega(\varepsilon_r - 1)\varepsilon_0} - \int_0^H G_z(z,z')J_z dl = E_i \qquad (4)$$

Equation (4) is the governing equation used in the MOM in this paper.

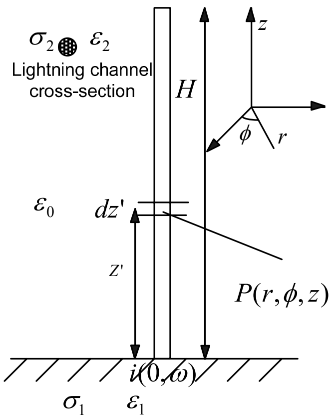


Fig. 1. Electromagnetic model of lightning return-stroke.

## III. Evaluation of Sommerfeld Integral by using Exact Image Theory

The most important prerequisite for a successful MOM is to obtain the Green's function for the specified problem. In the case of a return stroke, it is the solution of an infinitesimal dipole above a lossy half-space, which consists of following two components:

$$\mathbf{E}_s = \mathbf{E}_f + \mathbf{E}_d \qquad (5)$$

Where the scattering field $\mathbf{E}_s$ is divided into the field generated by an electric dipole in free space, $\mathbf{E}_f$, and the diffracted field $\mathbf{E}_d$. The z-component of the former has been found as

$$E_z^f = Idl\frac{e^{-jk_0R}}{4\pi R}jk_0\eta_0[(1-\frac{3j}{k_0R}-\frac{3}{k_0^2R^2})(\frac{z-z'}{R})^2 - (1-\frac{j}{k_0R}-\frac{1}{k_0^2R^2})] \qquad (6)$$

where the unknown line current $I$ on the segment $dl$ generate electric field at distance $R$ with an angular frequency $2\pi/k_0$.The intrinsic impedance of the free space is $\eta_0 = 120\ \pi$. As for the latter, it can be regarded as the superposition of some reflected plane waves

$$E_z(r,r') = -\frac{\eta_0 Idl}{4\pi k_0}\int_0^{+\infty}\frac{k_\rho^3}{k_z}J_0(k_\rho\rho)\Gamma_v e^{jk_z(z+z')}dk_\rho \qquad (7)$$

where; $J_0(k_\rho\rho)$ is the zero$^{th}$ order Bessel function of the first kind, $z$ and $z'$ are the z component of the source and observing point, respectively; $k_z$, $k_\rho$ are the components of the wave number in the cylindrical coordinate, i.e., $k^2 = k_z^2 + k_\rho^2$; $\Gamma_v$ is the Fresnel reflection coefficient.

The Laplace transformation of $\Gamma_v$ could be used to effectively simplify the computation of Sommerfeld integral, which forms the exact image theory [5].

## IV. Solving Integral Equation Based on FFT

Because MOM matrix is a full matrix, the method of straightforward factorization is prohibited when the number of unknowns exceeds some threshold. Instead of modifying the system matrix, iterative methods are preferred. During the updating process, the most time consuming step is the evaluation of the integral in (2), in which the unknown current **J** is replaced with intermediate solutions. Given translational mesh in this linear system, the convolution integral in (2) is proposed to be evaluated by using the fast Fourier transform as presented in (8) of [6], i.e.,

$$\mathbf{E}_s = dvFFT^{-1}[FFT(\mathbf{G})\cdot FFT(\mathbf{J})] \qquad (8)$$

Using the proposed approach, the computation time and required computer memory are greatly reduced.

## V. Numerical Example

According to [7], the based current of a lightning stroke is

$$i(0,t) = I_0(e^{-\alpha t} - e^{-\beta t}) \qquad (9)$$

where, $I_0 = 10$ kA, $\alpha = 3\times 10^4$ s$^{-1}$, $\beta = 10^7$ s$^{-1}$.

The upward velocity of the current along the channel is $1.1 \times 10^8$ ms$^{-1}$. The horizontal and vertical electric field radiated by such return stroke at the position $r$ = 200 m and $z$ = 10 m is calculated by using the proposed method. The comparison between the calculated results and the published ones of [7] is illustrated in Fig.2.

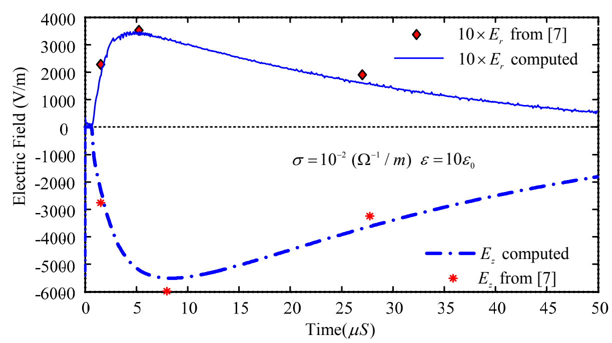


Fig. 2. The comparison of the horizontal and vertical electric field

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