High Frequency Multi-winding Transformers: from Numerical Simulation to Equivalent Circuits with Frequency-Independent RL Parameters

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***Abstract*—In this work, mathematically rigorous RL circuits of high frequency multi-winding transformers are identified using an original three-step numerical procedure. They are composed of frequency-independent coefficients and can therefore be introduced in classical circuit simulators. First, the Finite Element Method in 3D magnetodynamics, together with homogenization techniques which reduce the overall computational burden, are employed to extract frequency-dependent impedances and voltage gains. Then, the identified equivalent circuit is modified by removing the static resistances from the impedances, so as to obtain frequency-independent voltage gains, or coupling coefficients. An algorithm which is valid for any number of windings is proposed to that end. Finally, the frequency-dependent impedances are identified to Foster networks with constant elements using an optimization algorithm. An experimental validation with an impedance analyzer is proposed.**

***Index Terms*—Finite element methods, transformers, equivalent circuits, DC-DC power converters**

I. Introduction

Modern DC-DC power converters operate with high switching frequencies (from tens of kHz to several MHz), in order to reduce their volume and mass. This gives rise to frequency effects which are not correctly represented by classical models. In particular, at the considered frequencies, the multi-winding magnetic components comprised in the converters are subject to skin and proximity effects, dynamic magnetic losses in the ferrite core, and resonances due to the parasitic capacitances.

An appropriate use of numerical methods, such as the Finite Element Method (FEM), permits to capture these frequency effects. However, it is crucial to study the behaviour of the magnetic component in its entire electrical environment (*i.e.* within the whole power converter). To that end, a direct coupling between the FEM and the electrical circuit equations is feasible [1], but the resulting computational burden is very high, which is critical in the framework of a system approach. On the other hand, lumped parameter models consist in a more compact representation of the components, well-adapted to the introduction in circuit simulators for instance.

In this work, we focus on the identification of the RL part of the equivalent circuits using numerical methods (the capacitive part is for instance addressed in [2]). The literature has been abundant concerning that topic during the recent years. In [3] for instance, RLC circuits of multi-winding transformers are extracted using the FEM. However, the proposed circuits are not mathematically rigorous, in the sense that they do not comprise the exact number of parameters required to model all the coupling phenomena. In [4], a rigorous formalism for the construction of equivalent circuits of such components is described. The identification using experimental measurements has been proposed by the same author in [5]. However, the physical component is not always available, as in a research and development phase for instance. For simple geometries, such as planar transformers, analytical formulae can be employed to derive the circuits [6]. But the case of wound components is more complicated, and requires the use of the numerical tool.

In this paper, we propose a full numerical procedure to extract the equivalent RL circuits described in [4] for multi-winding magnetic components. The obtained circuits are made of constant elements, and can therefore be directly included in circuit simulators. Such a procedure has never been adressed in the literature to the best of the authors knowledge. It can be decomposed in three steps. First, the FEM in 3D magnetodynamics, together with homogenization techniques which reduce the overall computationnal burden, are employed to extract frequency-dependent impedances and voltage gains (section II-A). Then, the identified equivalent circuit is modified by removing the static resistances from the impedances, so as to obtain frequency-independent voltage gains, or coupling coefficients (section II-B). Finally, the frequency-dependent impedances are identified to Foster networks with constant elements using an optimization algorithm (section II-C). The procedure is illustrated and experimentally validated on a 3-winding ETD34 transformer in section III.

II. Three-step numerical procedure

A. Acquisition using the FEM

In the case of an n-winding component, the first step consists in using the FEM in 3D magnetodynamics in the frequency domain so as to extract n impedances $\underline{Z}'_k(f)$ and $n(n-1)/2$ voltage gains $\underline{\eta}'_{kl}(f)$, with f the frequency :

$$\underline{Z}'_k(f) = \frac{\underline{U}_k}{\underline{I}_k}, \begin{cases} k = n & : \quad \underline{I}_n = 1,\ \underline{I}_{1,\dots,n-1} = 0 \\ k < n & : \quad \underline{U}_n = 0,\ \underline{I}_{l=k} = 1,\ \underline{I}_{l\neq k} = 0 \end{cases} \tag{1}$$

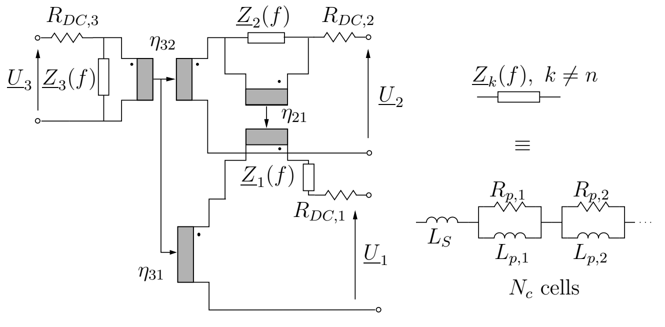
$$\underline{\eta}'_{kl}(f) = \frac{\underline{U}_l}{\underline{U}_k}, \begin{cases} k = n & : \quad \underline{I}_n = 1,\ \underline{I}_{1,\dots,n-1} = 0 \\ k < n & : \quad \underline{U}_n = 0,\ \underline{I}_{m=k} = 1,\ \underline{I}_{m\neq k} = 0 \end{cases} \tag{2}$$

In equations (1) and (2), $\underline{U}_k$ ($\underline{I}_k$) is the voltage (respectively the current) of winding k. We can see that the parameters are directly obtained by performing open and short-circuit tests numerically on the transformer. A 3D $\boldsymbol{a}-v$ formulation which enables a coupling with circuit relations is employed to that end [7]. Note that no further numerical integration is needed to obtain the parameters, as it is the case in [3].

At the considered frequencies, skin and proximity effects appear in the winding conductors. Very fine meshes are therefore required in order to get an acceptable accuracy. Here, a winding homogenization technique, based on [8] and extended to 3D cases, is used to reduce the computationnal burden. A complex permeability model is adopted for the ferrite core, which is also considered as a massive conductor.

B. Extraction of the static resistances

The second step consists in extracting the static resistances $R_{DC,k} = \underline{Z}'_k(f = 0)$ from the $\underline{Z}'_k(f)$ coefficients. It can be shown that the resulting equivalent circuit is characterized by n frequency-dependent impedances $\underline{Z}_k(f)$ and $n(n-1)/2$ frequency-independent coupling coefficients η_{kl}, as illustrated on Fig.1(a) for $n = 3$. These coefficients are computed using an original algorithm valid for any number of windings, which will be described in the full paper.



(a) (b)

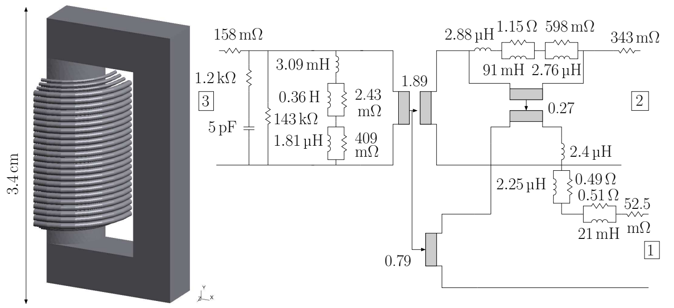
Figure 1: (a) Equivalent circuit obtained after the extraction of the static resistances, for $n = 3$, and (b) identification of frequency-dependent impedances to Foster networks.

C. Identification to Foster networks with constant elements

The frequency-dependent impedances $\underline{Z}_{k<n}(f)$ are replaced by a serial inductance and a cascade of parallel RL cells (or Foster network, see Fig.1(b)), as proposed in [5]. An optimization algorithm which aims at minimizing the square of the difference in modulus between the frequency-dependent impedance and the constant element network is employed. For the magnetization impedance $\underline{Z}_n(f)$, serial RC cells are added in parallel of the Foster network so as to model the dynamic magnetic losses, as it will be shown in the full paper.

III. TEST CASE: 3-WINDING ETD34 TRANSFORMER

The full procedure is applied to a 3-winding ETD34 transformer in ferrite PC90 (see Fig.2(a)). The corresponding frequency-independent RL circuit can be observed on Fig.2(b).



(a) (b)

Figure 2: (a) 3-winding ETD34 test transformer (quarter of the geometry), and (b) its frequency-independent RL circuit.

Without homogenization, the FE problem comprises 917776 dofs for a single frequency, and requires the use of parallelized solvers on a cluster. On the other hand, the homogenized model comprises 89123 dofs only, and can be handled with sequential direct solvers on a desktop PC. In the latter case, the full three-step procedure takes 31 min on a 2.7 GHz processor with 4Gb of RAM. The time consumed by the second and third steps is neglectible compared to the first phase involving FE computations. An experimental validation with an impedance analyzer will be proposed in the full paper. It will show that the RL circuit is valid up to 250 kHz. Beyond that limit, the resonances due to parasitic capacitances begin to appear.

REFERENCES

[1] G. Meunier, Y. Le Floch and C. Guérin, “A nonlinear circuit coupled t-t0- formulation for solid conductors,” in IEEE Transactions on Magnetics, 39(3), pp. 1729-1732, 2003.

[2] J. Biela, “Using transformer paraitics for resonant converters - a review of the calculation of the stray capacitances of the transformers,” in IEEE Transactions on Industry Applications, 44(1), pp. 223-233, 2008.

[3] R. Asensi, R. Prieto, J. Cobos and J. Uceda, “Modeling high frequency multi-winding magnetic components using finite-element analysis,” in IEEE Transactions on Magnetics, 43(10), pp. 856-863, 2007.

[4] J.-P. Kéradec, “Transformateurs HF à n enroulements: schémas à constantes localisées,” Techniques de l’Ingnieur, vol. D-3058, 2008.

[5] J.-P. Kéradec, “Transformateurs HF à n enroulements: identification expérimentale,” Techniques de l’Ingnieur, vol. D-3059, 2009.

[6] A. Besri, X. Margueron, J.-P. Kéradec and B. Delinchant, “Wide frequency range lumped element equivalent circuit for HF planar transformer,” in Proc. of the Power Electronics Specialists Conference (PESC), pp. 766-772, 2008.

[7] P. Dular, F. Henrotte, W. Legros, “A General and Natural Method to Define Circuit Relations Associated with Magnetic Vector Potential Formulations,” in IEEE Trans. Mag., 35(3), pp. 1630-1633, 1999.

[8] J. Gyselinck and P. Dular,“Frequency-Domain Homogenization of Bundles of Wires in 2D Magnetodynamic FE Calculations,” in IEEE Transactions on Magnetics, 41(5), pp. 1416-1419, 2005.