Time Domain Absorbing Boundary Terminations for Waveguide Ports based on State Space Models

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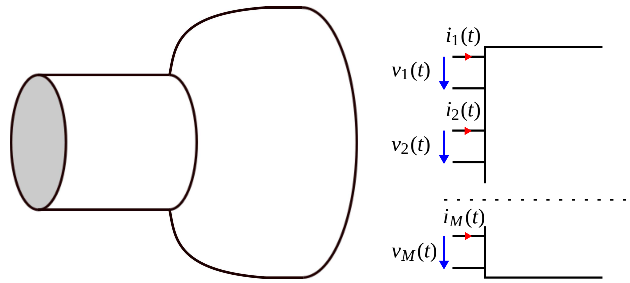
***Abstract*—Absorbing boundary conditions for waveguide ports in time domain are important elements of transient approaches to treat RF structures. A successful way to implement these termination conditions is the decomposition of the transient fields in the absorbing plane in terms of modal field patterns. The absorbing condition is then accomplished by transferring the wave impedances (or admittances) of the modes to time domain, which leads to convolution operations involving Bessel functions and integrals over Bessel functions. This article presents a new alternative approach: the convolution operations are approximated by appropriate state space models whose system responses can be conveniently computed by standard integration schemes. These schemes are indispensable for transient simulations anyhow.**

***Index Terms*—Waveguide boundary condition, modal wave absorption, time domain analysis, modal analysis.**



(a) (b)

Figure 1. (a) Segment of a (not necessarily) circular waveguide port of a 3D RF structure. The grey facet indicates the absorbing waveguide boundary. (b) Equivalent circuit of the waveguide port, where only M (of an infinite number of) port modes are indicated by their modal voltages $v_m(t)$ and currents $i_m(t)$.

I. Introduction

The investigation of closed RF structures with waveguide ports in time domain is a standard task in Computational Electromagnetics. Often, for this purpose boundary conditions at the waveguide ports are required ensuring waves incident on the ports from inside of the RF structure are not scattered back into the structure. In the literature a large variety of methods (see e.g. [1], [2], [3]) is discussed to tackle this problem. This article presents a new approach to construct multimodal waveguide port boundary termination conditions for time domain computations employing state space models (SSM). The presented method is based on the transform of the well-known characteristic impedances of port modes in frequency domain to time domain, as it is proposed in [4], [5]. However, in contrast to [4], [5] the presented modal absorbing boundary condition (MABC) does not require to evaluate convolution integrals involving Bessel functions. Instead, the convolution operations are approximated by suitable state space models, whose system responses are conveniently computable by means of standard integration schemes. This is a key benefit of this approach since such integration schemes are a central element of time domain simulations. Thus, the method can be easily implemented. Moreover, it can be employed for arbitrary waveguide cross sections and works below and above cutoff frequencies of the respective waveguide modes.

II. General Theory

To convey the basic idea for the proposed termination condition, a waveguide port which is connected to a RF structure via a short waveguide with constant cross section is considered in Fig. 1(a). Facet $\partial\boldsymbol{\Omega}_{\mathrm{wg}}$ (grey area) denotes the absorbing waveguide boundary. Waves travelling from the RF structure towards the absorbing boundary are not reflected. It is commonly known that the transient transverse electric and magnetic fields in the plane $\partial\boldsymbol{\Omega}_{\mathrm{wg}}$ (which is enclosed by PEC walls) can be described by means of frequency and time independent orthonormalized mode patterns $\mathbf{L}_{t,m}(\mathbf{r}_t)$:

$$\mathbf{E}_t(\mathbf{r}_t, t) = \sum_{m=1}^{\infty} \mathbf{L}_{t,m}(\mathbf{r}_t)\, v_m(t), \tag{1}$$

$$\mathbf{H}_t(\mathbf{r}_t, t) = \sum_{m=1}^{\infty} \mathbf{n}_z \times \mathbf{L}_{t,m}(\mathbf{r}_t)\, i_m(t), \tag{2}$$

where $v_m(t)$ and $i_m(t)$ are referred to as modal voltages and currents and $\mathbf{n}_z$ is the normal vector of $\partial\boldsymbol{\Omega}_{\mathrm{wg}}$ pointing into the structure. The transverse spatial coordinates are denoted by $\mathbf{r}_t \in \partial\boldsymbol{\Omega}_{\mathrm{wg}}$. The patterns $\mathbf{L}_{t,m}(\mathbf{r}_t)$ are determined by solving the 2D Helmholtz equation on $\partial\boldsymbol{\Omega}_{\mathrm{wg}}$. Moreover, the patterns regard TE as well as TM waveguide modes. From (1) and (2) it is obvious that an infinite number of modes is needed for the field construction. However, only a finite number of modes with cutoff frequency ω_{co} in or below the frequency interval of interest can principally propagate through the waveguide of constant cross section, depicted in Fig. 1(a). The remaining modes are evanescent modes with cutoff frequency ω_{co} above the frequency interval of interest. These modes exponentially decay along the waveguide with decay rate depending on ω_{co}. Thus, all modes above cutoff and only the first set of modes below cutoff need to be considered in (1) and (2).

Consequently, it is sufficient to employ only this finite number of "accessible modes" [4, p. 476] for the transverse field expansion. Fig. 1(b) shows the equivalent circuit for the waveguide port. M mode patterns and their respective transient modal voltages $v_m(t)$ and currents $i_m(t)$ are indicated. The exact "infinite guide" [4, p. 476] modal termination conditions for the modal voltages and currents are well-known in frequency domain:

$$\underline{I}_m(s) = \underline{Z}_{0,m}^{-1}(s)\, \underline{V}_m(s) = \underline{G}_{0,m}(s)\, \underline{V}_m(s), \tag{3}$$

where $s = j\omega$ is the complex angular frequency and $\underline{V}_m(s)$ and $\underline{I}_m(s)$ are the modal voltages and currents in frequency domain. $\underline{Z}_{0,m}(s)$ and $\underline{G}_{0,m}(s)$ denote the modal characteristic wave impedances and admittances. Relation (3) emulates the short waveguide with constant cross section (refer to Fig, 1(a)) to be infinitely long such that there is no reflection from $\partial\mathbf{\Omega}_{\mathrm{wg}}$. The modal characteristic wave impedances and admittances are different for TE and TM port modes:

$$\underline{G}_{0,m}^{\mathrm{TE}}(s) = \frac{1}{\underline{Z}_{0,m}^{\mathrm{TE}}(s)} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{\sqrt{s^2+\omega_{\mathrm{co},m}^2}}{s}, \tag{4}$$

$$\underline{G}_{0,m}^{\mathrm{TM}}(s) = \frac{1}{\underline{Z}_{0,m}^{\mathrm{TM}}(s)} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{s}{\sqrt{s^2+\omega_{\mathrm{co},m}^2}}, \tag{5}$$

where $\sqrt{\varepsilon_0/\mu_0}$ is the free space admittance and $\omega_{\mathrm{co},m}$ the cutoff frequency of the m-th mode. For each mode the transient electric field at the termination corresponds to a transient magnetic field which has to be assigned in the plane $\partial\mathbf{\Omega}_{\mathrm{wg}}$ to accomplish the absorbing condition. Therefore, (3) is transformed into time domain. This yields the convolution

$$i_m(t) = g_{0,m}(t) * v_m(t), \tag{6}$$

with the inverse Laplace transform $\mathcal{L}^{-1}\{\underline{G}_{0,m}(s)\}(t) = g_{0,m}(t)$. Both inverse Laplace transforms corresponding to (4) and (5) can be found in [4], [5], [6]. Bessel functions and integrals over Bessel functions are involved in $g_{0,m}(t)$. Alternatively, the currents $i_m(t)$, which are dependent on the voltages $v_m(t)$ can be approximated by a state space model

$$\frac{\partial}{\partial t}\mathbf{x}(t) = \omega_{\mathrm{co},m}\,\mathbf{A}\,\mathbf{x}(t) + \omega_{\mathrm{co},m}\,\mathbf{B}\,v_m(t), \tag{7}$$

$$i_m(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,v_m(t), \tag{8}$$

in combination with standard integration schemes which are needed anyhow while performing transient simulations. In fact, two different sets of state space models need to be constructed: one set for TE modes $\{\mathbf{A}^{\mathrm{TE}}, \mathbf{B}^{\mathrm{TE}}, \mathbf{C}^{\mathrm{TE}}, \mathbf{D}^{\mathrm{TE}}\}$ and one set for TM modes $\{\mathbf{A}^{\mathrm{TM}}, \mathbf{B}^{\mathrm{TM}}, \mathbf{C}^{\mathrm{TM}}, \mathbf{D}^{\mathrm{TM}}\}$, respectively. These matrices are created by approximating the modal characteristic admittances with a cutoff frequency of $\omega_{\mathrm{co}} = 1\,\mathrm{rad/s}$ using a fraction:

$$\underline{G}_0(s) \approx \frac{a_0 + a_1 s + \ldots a_N^N}{b_0 + b_1 s + \ldots + b_N^N} = \mathbf{C}(s\,\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}. \tag{9}$$

One way to determine the coefficients a_n and b_n is the application of a Padé approximation about the point $s_0 = 1$. Once these coefficients are known, the state matrices are filled such that the frequency domain transfer function of (7) and (8) equals the rational function (r.h.s of (9)). Obviously the quality of the approximation is improved if the order N of the rational function is increased. Yet, large N lead to large state matrices, since $\mathbf{A} \in \mathbb{R}^{N\times N}$, $\mathbf{B} \in \mathbb{R}^{N\times 1}$, $\mathbf{C} \in \mathbb{R}^{1\times N}$, $\mathbf{D} \in \mathbb{R}^{1\times 1}$.

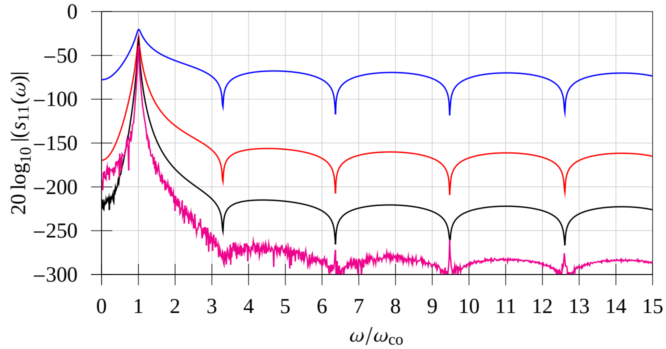


Figure 2. Reflection factor $s_{11}(\omega)$ of a TE mode with $\omega_{\mathrm{co}} = 1\,\mathrm{rad/s}$ in a waveguide with length $L = c_0/\omega_{\mathrm{co}}$, where c_0 is the speed of light in vacuum. The waveguide is matched at both ends with the presented termination condition. The calculation is performed based on analytically known field distributions. The blue curve plots the artificial reflection of the MABC for $N = 6$, red for $N = 12$, black for $N = 16$ and magenta for $N = 20$.

III. Numerical Validation

The validation of the scheme is performed in frequency domain as this allows for the comparison of the results with analytical formulas. Nonetheless, the marginal stability of (7) is checked via the eigenvalues λ_i of the state matrix $\mathbf{A}$, i.e. $\Re(\lambda_i) \leq 0\ \forall i$. To determine the quality of the MABC based on SSMs, the analytical field distribution of a TE mode in a waveguide with constant cross section is considered. The approximated termination condition (r.h.s of (9)) is assigned to voltages and currents of the modes at both waveguide ends. Accordingly, the reflection coefficients of the waveguide (which have to be zero for an ideal absorbing boundary condition) are computed for different N, see Fig. 2. For $N = 20$ the artificial reflection falls below $-250\,\mathrm{dB}$ above the cutoff frequency. Moreover, all plots have in common that the maximal reflection is at $\omega = \omega_{\mathrm{co}}$. This reflection is almost invariant with respect to N.

IV. Summary and Conclusions

The presented MABC avoids the explicit calculation of convolutions with Bessel functions. Sufficiently far away from the cutoff frequency of the modes an artificial reflection in the order of the numerical noise is achieved. However, the maximal reflection at the cutoff frequency is independent of the approximation order N and stays at $\approx -30\,\mathrm{dB}$.

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