Wide-band electromagnetic time reversal : selecting the instant of focus for scatterer localization

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***Résumé*—Time Reversal (TR) techniques are widely used in ElectroMagnetic (EM) imaging and localization of buried objects. In a wide-band study, the localization of an object requires the determination of an instant of focus that is, the instant when the energy of the back-propagated scattered fields reaches its location. We recall a new criterion (recently introduced) for choosing this instant of focus and we compare it to the minimum entropy criterion existing in the literature [1]. We present the results obtained for the localization of underground regions, in synthetic 2D configuration : it is shown that the proposed criterion can give good results when the minimum entropy criterion fails. We also present an experimental validation for 3D objects.**

I. INTRODUCTION

In non-dissipative media, the solution of Maxwell's equations is time symmetric. This time reversibility can be exploited by the fact that a wave emitted by a source may turn back and return to its original source enabling us to locate it.

Time-domain approaches for source localization can be developed in frequency [2] or time domain [3]. An issue, specific to time-domain approaches, consists in finding the instant when the back-propagated fields distribution could allow us to locate the sources. In the literature, the so-called minimum entropy criterion is used to select this instant of focus [4]. However, it is sensitive to the frequency band and does not always give relevant results.

We refer to the criterion recently introduced in [5] for choosing the instant of focus. We first expose results obtained from a fully synthetic 2D configuration modeled with Finite Integration Technique (FIT). Then, we provide an experimental validation in which measured signals are back-propagated into the numerical model.

II. NUMERICAL MODEL FOR WIDE-BAND TR

We are interested in scatterers considered as secondary sources when excited by an EM wave [6]. We model the phenomena with FIT, which is based on the discretization of the integral form of Maxwell's equations on two dual meshes [7]. The FIT provides a general formalism for finite difference and finite elements methods [8]. Depending on the type of mesh needed for the simulations, the constitutive matrices can be built using either the Galerkin or the orthogonal method in order to make the computations more efficient [9]. In our model, we use unstructured meshes to model direct problems and orthogonal grids for the back-propagation problem.

Once the back-propagated fields are computed within a time interval and in a search area, the EM energy density ρ is calculated in each element n_j of the homogeneous orthogonal grid. The instant (of focus) maximizing the inverse of the entropy, applied to the energy density, is given by

$$\psi(t) = \frac{\sum_j \rho_j^2(t)}{\left[\sum_j \rho_j(t)\right]^2}. \tag{1}$$

The criterion which we introduce here is based on the decomposition (for each instant t) of the EM energy density cartography on a number K of layers S_k

$$n_j \in S_k(t) \Leftrightarrow \frac{K-k}{K}\rho_{max}(t) < \rho_j(t) \leq \frac{K-k+1}{K}\rho_{max}(t). \tag{2}$$

An adequate weighted sum

$$\phi_K(t) = \sum_{k=1}^{K} \frac{kN_k(t)}{KN} \tag{3}$$

is performed on the volumes $N_k(t) = |S_k(t)|$ of these layers and K is stretched to infinity. Finally, the instant of focus is chosen as the one that maximizes the following function

$$\phi_\infty(t) = 1 - \frac{\sum_{j=1}^{N} \rho_j(t)}{N\rho_{max}(t)}. \tag{4}$$

III. APPLICATIONS

A. Localization of air regions in heterogeneous dielectric medium

Let us consider two air ellipsoidal regions in an heterogeneous dielectric medium. The scene is illuminated by a pulsed electric dipole, Fig. 1 (left). The incident field is computed in the homogeneous medium considering the mean permittivity in no air regions. The signals corresponding to the scattered field are computed on the dipoles forming the TR Mirror (TRM). At the back-propagation step, these signals are time-reversed and injected by the TRM into the homogeneous medium without air regions.

The function ϕ_∞ and ψ are computed on a well-chosen interval of time and their behavior is shown in Fig. 1 (right). The

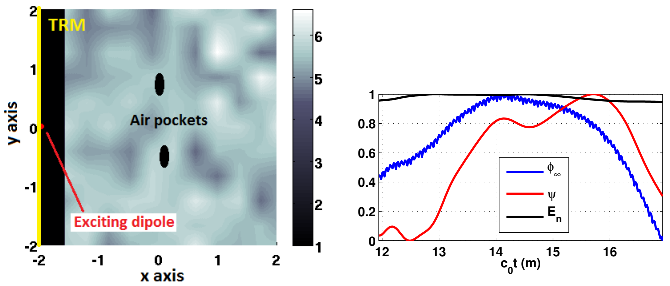

FIGURE 1. Distribution of the relative permittivity (left) and behavior of the normalized ϕ_∞, ψ and E_n (the EM energy in the search area).

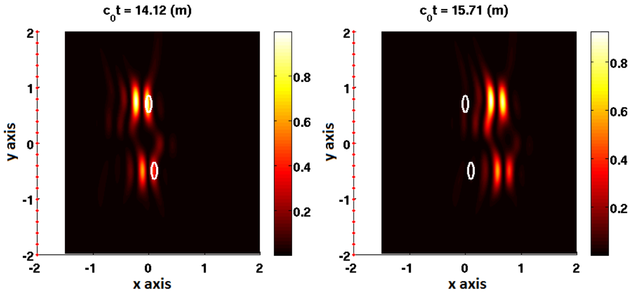

FIGURE 2. Distribution of EM density of energy at the instant given by ϕ_∞ (left) and ψ (right).

criterion ϕ_∞ gives an instant of focus t so that $c_0t = 14.12$ m and the inverse of entropy reaches its maximum at $c_0t = 15.71$ m. The instant given by the latter does not yield a correct localization contrarily to the result given by the introduced criterion, Fig. 2. However, ψ reaches a local maximum at $c_0t = 14.12$ m.

B. Experimental validation

Measurements of the electric field scattered by metallic objects (plate and stem) are made in the frequency band $[0.5\,\text{GHz}\;3\,\text{GHz}]$ in an anechoïc chamber using a vector network analyzer. A synthetic pulse is considered and the corresponding time responses are calculated at 9×9 measurement points, Fig. 3. Then, these signals are back-propagated into the numerical model.

The behavior of ϕ_∞ and ψ for the case of each object (plate or stem) is depicted in Fig. 4. The two functions reach their maximum at the same time for each object. The corresponding distributions of the EM density of energy are given in Fig. 5. The two criteria provide correct results in each case.

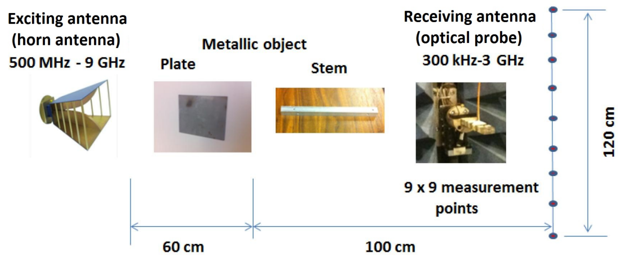

FIGURE 3. Experimental setup description.

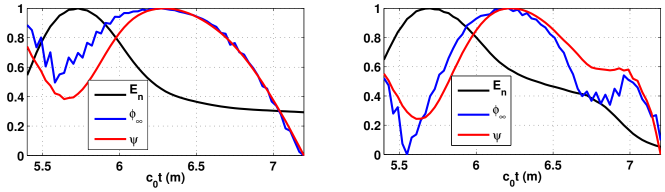

FIGURE 4. Behavior of ϕ_∞ and ψ in the case of the plate (left) and stem (right).

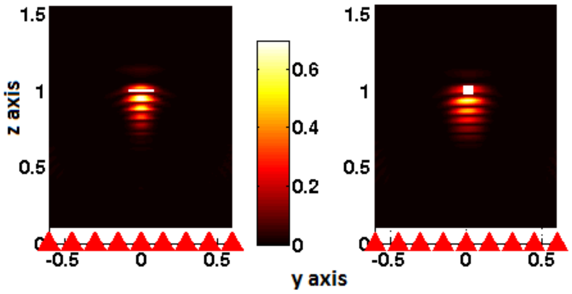

FIGURE 5. Distribution of EM energy density in plane $x = 0$ at instant of focus, for the plate (left) and the stem (right).

IV. CONCLUSION

We have contributed to the study of wide-band time reversal by the introduction of a new criterion for choosing the instant of focus. This criterion shows satisfactory results in both synthetic and experimental configurations. It is complementary to the criterion of minimum entropy and allows for a better identification of the instant of focus in a number of configurations.

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