# **Numerical Dynamic Strong Coupled Model of Linear Magnetostrictive Actuators**

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**Abstract —A numerical dynamic and magnetomechanical strong coupled model of magnetostrictive actuators was founded. The model includes the magneto-mechanical coupling term, which reflects the strong coupling characteristic of magnetostrictive actuators. With this model, the dynamic magnetic and the mechanical problems can be solved simultaneously. According to the proposed model and the measured magnetic characteristic of magnetostrictive material, the relation between the magnet field and the output displacement of the actuator was calculated based on the finite element method. A comparison between the calculating results and the experimental ones for a actuator was carried out and it was found that they were in agreement well. This indicates that the numerical dynamic magnetomechanical strong coupled model can be used to the design and control of magnetostrictive actuators.**

### I. INTRODUCTION

The Magnetostrictive Material, such as Terfenol-D, is a new type of functional materials which can produce large deformation when it is in magnetic field. For its high dynamic strain and high energy coupling factor, the Magnetostrictive Material is more suitable to be used in actuator than other smart material such as piezoceramics and electrostrictives. As a novel type of device which transfers electro-magnetic energy to mechanical energy, magnetostrictive actuator can be used in micro-orientation system, precision mechanical process, etc.

Magnetostrictive actuator using Terfenol-D exhibits nonlinear behavior and magnetomechanical coupling characteristic. The mechanical state affects the magnetic state and vice versa. These make it difficult to model the actuator which incorporates both the magnetomechanical behaviors and the dynamic characteristic. Linearized models, such as Claeyssen [1], are the most common method of modeling magnetostrictive materials. These models present a high computational efficiency and require small number of input parameters. However, more powerful models must be applied for strongly nonlinear systems. Nonlinear models for Terfenol-D based on the Finite Difference Method (FDM) have been developed by Engdahl and Kvarniö [2]. These models require the nonliear material characteristics as numerical input. When modeling a magnetostrictive system, including the magnetostrictive material, the magnetizing coil, the magnetic circuit, etc., the FDM shows some difficulties concerning the boundaries and interfaces between the different parts of the system. To overcome these obstacles, the Finite Element Method (FEM) has been used in calculating magnetostrictive problems and some

achievements have been gotten. Much work have been done using FEM through the weak coupling model or the strong coupling model [3]-[4]. However, these FEM models were suitable to simulate the static state or quasistatic state of the magnetostrictive materials or actuators. There have some limitations when they were used in dynamic system. In this paper, based on FEM, a dynamic and magnetomechanical strong coupled model of magnetostrictive actuators is proposed.

## II. NUMERICAL DYNYMIC STRONG COUPLED MODEL

When AC current flows in the exciting winding of the actuator, the output displacement is a dynamic variable and the actuator together with its load is a dynamic system. Based on Hamilton principle of minimal potential energy, equation (1) is correct for a dynamic system.

$$
\delta \int_{t_1}^{t_2} (T^e - U^e - W^e) dt = 0 \tag{1}
$$

where  $W^e$ ,  $T^e$  and  $U^e$  denote the work of external forces, the kinetic energy and the potential energy of the element respectively.

For the designed magnetostrictive actuator, the kinetic energy of the element, including mechanical kinetic energy and magnetic energy, can be expressed by

$$
T^{e} = \int_{e} \frac{1}{2} \rho \dot{u}^{T} \dot{u} dV + \int_{e} H \cdot dB - \int_{e} J \cdot dA \tag{2}
$$

where superscript *T* expresses transposed matrix,  $\rho$  and *V* denote the density of the material and the volume of the

element respectively. 
$$
u = \begin{bmatrix} u_r \\ u_z \end{bmatrix}
$$
 denotes displacement

vector. 
$$
\dot{u} = \begin{bmatrix} \frac{du_r}{dt} & \frac{du_z}{dt} \end{bmatrix}^T
$$
 denotes velocity vector,

$$
B = \begin{bmatrix} B_r \\ B_z \end{bmatrix}, H = \begin{bmatrix} H_r \\ H_z \end{bmatrix}, J \text{ and } A \text{ denote the magnetic flux}
$$

density, the magnetic field intensity, the current density and the magnetic vector potential respectively. As for axissymmetric field, there has  $A_r = A_1 = 0$ ,  $A_\theta = A$ ,

$$
J_r = J_z = 0 \, , \, J_\theta = J \, .
$$

The work of external forces is given by

$$
W^e = \int_e f^e \cdot u dV + \int_s f^s \cdot u ds \tag{3}
$$

Where  $f^e$ ,  $f^s$  are the external volume force and the external surface force, respectively. *s* is the boundary of the external surface force.

The potential energy of the element is given by

$$
U^e = \int_e \sigma \cdot d\varepsilon \tag{4}
$$

where  $\sigma = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  $\rfloor$  $\left| \frac{\sigma_r}{\sigma} \right|$ L  $=$ *z r* σ  $\sigma = \left| \frac{\sigma_r}{\sigma} \right|$  and  $\varepsilon = \left| \frac{\varepsilon_r}{\sigma} \right|$  $\rfloor$  $\left| \begin{array}{c} \mathcal{E}_{r} \\ \mathcal{E}_{r} \end{array} \right|$ L  $=$ *z r* ε  $\varepsilon = \begin{vmatrix} \varepsilon_r \\ \varepsilon_r \end{vmatrix}$  are stress tensor and strain

tensor respectively. The linear constitutive piezomagnetic equations for the magnetostrictive material are given as (5)

$$
\begin{cases}\n\sigma = D\varepsilon - D dH \\
B = d\sigma + \mu H\n\end{cases}
$$
\n(5)

where  $D = \frac{E}{1-\alpha^2}\begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix}$  $\mathbf{r}$  $=\frac{E}{1-\alpha^2}\begin{bmatrix}1 & \alpha \\ \alpha & 1\end{bmatrix}$  $1-\alpha^2$   $\alpha$ α α  $D = \frac{E}{\epsilon} \begin{vmatrix} 1 & \alpha \\ 1 & \beta \end{vmatrix}$  is the elastic modulus matrix,

E is the Young modulus and *α* is Poisson's ratio,

$$
d = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}
$$
 is the magnetoelastic coupling coefficients

matrix for *r r H*  $d_{11} = \frac{\partial S}{\partial t}$ ∂  $\sum_{11}$  =  $\frac{\partial S_r}{\partial H}$  and *z z H*  $d_{22} = \frac{\partial S}{\partial x_i}$ ∂  $\frac{\partial S_z}{\partial H}$ . Here the axial

and the radial components are only taken into account, the magnetic filed component and the stress component are the same direction. Introducing (5) into (4), gives

$$
U^e = \frac{1}{2} \int_e \varepsilon D \varepsilon dV - \int_e D dH \varepsilon dV \tag{6}
$$

where the second term is the magneto-mechanical coupling term, which reflects the strong coupling characteristic of the actuator.

Introduce  $(2)$ ,  $(3)$  and  $(6)$  into  $(1)$ , then, the dynamic model for the actuator system can be expressed as

$$
\delta \int_{t_1}^{t_2} \{ \int \frac{1}{2} \rho u^T u dV + \int \limits_{e} H \cdot dB - \int \limits_{e} J \cdot dA
$$
  
+ 
$$
\int \limits_{e} f^e \cdot u dV + \int \limits_{s} f^s \cdot u ds
$$
  
+ 
$$
\frac{1}{2} \int \limits_{e} \epsilon D \epsilon dV - \int \limits_{e} D dH \epsilon dV \} dt = 0 \quad (7)
$$

Through the finite element discretizing and variational differential calculating, the kinetic equation of the system from (7) can be obtained. With the magnetomechanical strong coupling model, the magnetic and the mechanical problems can be solved simultaneously by using FEM, and the dynamic input-output numerical value can be obtained.

#### III. COMPUTED AND EXPERIMENT RESULTS

The above-mentioned procedure has been realized by a computer program. According to the experimental curve of the flux density and the magnetic field intensity for the giant magnetostrictive materials, the Young modulus and the Poisson's ratio in [5]-[6]. The relation between the exciting magnetic field and the output displacement for the

actuator was calculated. The calculated results were compared with experimental ones, which were shown in Fig.1. From Fig.1, It is obviously that the actuator exhibits dynamic hysteresis when it is excited by AC applied field. The amount of magnetic hysteresis is well predicted by the proposed model, the slope of the ouput displacement is characterized by the model. The calculated results are in a good agreement with the experimental ones. Deviation between experiment and model at high magnetic field may be due to inaccurate characterization of the magnetic bias field.The maximum deviation does not exceed 3.7%. In spite of some deviations, the model provides a reasonable characterization of the dynamic experimental displacement.



Fig. 3. Displacement vs. magnetic field for 35.30kA/m magnetic bias, 17.65kA/m applied field, 50Hz frequency excitation. ( \* Experimental results,  $\longrightarrow$  Calculating results )

#### IV. CONCLUSION

Based on Hamilton principle of minimal potential energy, a numerical Dynamic model for magnetostrictive actuators was presented. The relation between the exciting magnetic field and the output displacement for the actuator was calculated by means of FEM. A comparison between the calculated results and experimental ones for the actuator was carried out and it was found that they were in agreement well. Therefore it is of great practical value to design and optimize the structure and control magnetostrictive devices.

#### V. REFERENCES

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