Fast and Accurate Prediction of Reverberation Chambers' Resonant Frequencies Using Time-Domain Integral Equation and Matrix Pencil Method

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Abstract — Unstirred frequencies of a reverberation chamber (RC) are attributed to its discrete mode distributions. Transient analysis is usually preferred in predicting an RC's mode distributions. For its geometrical flexibility, the time-domain integral equation (TDIE) is suitable for transient analysis of RCs containing stirrers of arbitrary shape. Due to the resonant nature of an RC, a large number of marching steps are usually required for the accurate prediction of resonant frequencies. This paper adopts the TDIE to efficiently obtain the early time response of an RC, and utilizes the matrix pencil method to accurately calculate its resonant frequencies. The proposed method can accurately predict an RC's resonant frequencies using the early time information, and it can shorten the time-consuming TDIE simulations.

I. INTRODUCTION

Reverberation chamber (RC) modeling recently attracted much interest for its importance in evaluating new stirring methods and optimizing RC designs [1], [2]. In electromagnetic compatibility applications, the presence of unstirred frequencies is considered a limitation for RC's use. It has been demonstrated that unstirred frequencies were unavoidable in an RC, which could be ascribed to the presence of a discrete mode distribution [3]. In order to predict an RC's mode distribution, transient analysis is desirable because it can provide wideband information using one single simulation. Among various time-domain numerical methods, the time-domain integral equation (TDIE) [4] is preferred for RC modeling, because it can easily and accurately model stirrers of irregular shape. Although subcell techniques render geometrical flexibility for the finite difference time domain method, their convergence is not always guaranteed [5]. In order to obtain the frequency-domain information, fast Fourier transform (FFT) is usually applied to the time-domain signal. However, the FFT requires long-time information to obtain good frequency resolution. On the other hand, it is known that spectrum estimation techniques [6] are accurate in predicting resonant frequencies using short-time information.

This paper proposes to calculate resonant frequencies of an RC using the combination of TDIE and matrix pencil method (MPM). The time-domain signal is expanded using the damped sinusoids. With early time information obtained using the TDIE, angular frequencies of sinusoids are estimated using the MPM. These angular frequencies correspond to the resonant frequencies of an unstirred RC. In this case, the resonant frequencies can be calculated using very short time-domain information, and time-consuming TDIE simulations can then be avoided. Numerical results are presented to illustrate the efficacy and advantages of our proposed method.

II. FORMULATION OF TDIE AND MPM

The TDIE has been widely used in solving electromagnetic radiation and scattering problems [7], [8]. For RC modeling, the excitation is different from those in radiation and scattering problems. A transmitting antenna is usually placed at one corner of the RC. For simplicity, the transmitting antenna can be replaced by a current source $\vec{J}_0$. This is reasonable when the RC itself is the one of main concern. The radiated fields by the current source will then be the fields illuminating cavity walls and stirrers. Denote surfaces of cavity walls and stirrer as *S*. Along *S*, the following boundary condition should be satisfied at any time

$$\hat{t}(\vec{r})\cdot\left(\vec{E}^s(\vec{r},t)+\vec{E}^i(\vec{r},t)\right)=0, \quad \text{for } \vec{r}\in S,$$

where $\vec{E}^s$ is the electric field due to the induced current $\vec{J}_s$ on *S*, and $\vec{E}^i$ is the electric field from the current source $\vec{J}_0$. $\hat{t}(\vec{r})$ is an unitary vector tangential to *S* at $\vec{r}$. $\vec{E}^i$ can be calculated by an integral equation

$$\begin{aligned}\vec{E}^i(\vec{r},t) &= -\frac{\mu}{4\pi}\frac{\partial}{\partial t}\int_{S_0}\vec{J}_0(\vec{r}',t-R/c)\big/R\,dS' \\ &+\frac{1}{4\pi\varepsilon}\int_{S_0}\int_0^t\nabla'_s\cdot\vec{J}_0(\vec{r}',\tau-R/c)\big/R\,d\tau dS'\end{aligned},$$

where S_0 represent the domain of $\vec{J}_0$. $R=|\vec{r}-\vec{r}'|$, and $\vec{r}$ and $\vec{r}'$ are observation and source points, respectively. For calculating $\vec{E}^s$, one only needs to replace $\vec{E}^i$, $\vec{J}_0$, and S_0 in the above equation by $\vec{E}^s$, $\vec{J}_s$, and *S*, respectively. $\vec{J}_s$ is unknown, and it is then expanded using RWG and triangular bases in the spatial and temporal domains, respectively. By enforcing the boundary conditions on *S*, one can obtain an explicit marching on in time scheme for calculating $\vec{J}_s$ recursively. Once $\vec{J}_s$ is obtained, the total field in the RC can be calculated using $\vec{J}_s$ and $\vec{J}_0$. The resonant frequencies can then be extracted from the time-domain field information.

Since FFT requires long-time information for problems of resonant nature, this work utilizes the MPM to extract resonant frequencies from short-time information. For a time-domain signal $x(t)$, it is expanded as

$$x(t)=\sum_{n=1}^{N} a_n \exp[(-\alpha_n + j2\pi f_n)t],$$

where f_n is the nth resonant frequency, α_n is the attenuation constant of the nth resonant frequency, a_n is the expansion coefficient, and N is an integer chosen to be two times the number of resonant frequencies. Using the MPM in [4], f_n, α_n, and a_n can be estimated from early time information. The choice of N is crucial for accurate calculation of resonant frequencies. In [4], the total least squares MPM was proposed to estimate N. However, a criterion must be set by users in the least squares MPM, and different criteria may give different results. A simple and robust method to determine N is to get the rough spectrum using FFT of early time information. The number of peaks in the rough spectrum will be chosen as the number of resonant frequencies.

III. Numerical Examples

Numerical examples are presented to show the efficacy of the proposed method. Following [4] , the unit of time used here is light meter (lm), which is the time that the light takes to travel one meter. The RC considered has dimensions 1.7 m×2.5 m×1.2 m. It has a vertical five-paddle stirrer. All paddles are rectangular and have the same dimension of 0.2298 m×0.26 m. The angle between each two paddles is 115 degrees. The following modulated Gaussian pulse is used as the current excitation

$$\vec{J}(\vec{r},t)=\vec{J}_0(\vec{r})\frac{1}{\sqrt{\pi}}\exp\left[-\frac{c^2}{\sigma^2}(t-t_0)^2\right]\cos\left(2\pi f_c\left(t-t_0\right)\right),$$

where f_c is the center frequency, c is the speed of light, σ is used to adjust the bandwidth of the pulse, t_0 is the time shift to make the pulse negligible at t=0, and $\vec{r}$ is the position of the current source. In obtaining results presented in this work, $\vec{J}_0=\hat{x}$, f_c=0.15 GHz, $\vec{r}=0.15\hat{x}+0.15\hat{y}+0.15\hat{z}$, σ=1.5, and t_0=6 lm. Besides the RC with a vertical stirrer, another two cases are also considered. First, for verification purpose, the cavity without the vertical stirrer is analyzed, for which analytical results are available. Second, since the present formulation does not consider the loss, a small square aperture is made on one wall of the RC to account for the loss.

Fig. 1 illustrates the rough spectrum of E_x at the center of the cavity in the three cases. The spectrum is obtained by the FFT of early time response from 30 lm to 42.75 lm. Its frequency resolution is very low, and better resolution will require long-time information. The two peaks in Fig. 1 indicate that there are two resonant modes excited, and N should be chosen to be four in the MPM. Table 1 presents the resonant frequencies calculated using different approaches. The results are obtained with the time information from 30 lm to 36 lm, with time step size of 0.1 lm. The proposed method is validated using analytical prediction in the case of an empty cavity. The total least squares MPM fails to predict the expected resonant frequencies. This is reasonable because the total least squares MPM can not estimate the number of resonant modes accurately. With the exact number of resonant modes, the proposed method can generate accurate results in all cases. In calculating the resonant frequencies, the proposed method only requires time-domain information up to 42.75 lm. More time-domain information would be required if FFT were used, which means that more time-consuming TDIE simulations would be required.

IV. References

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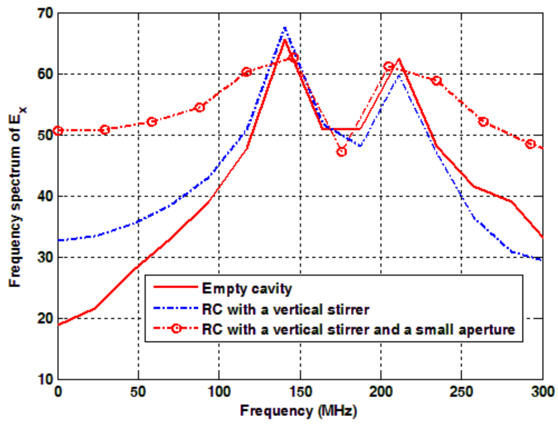

Fig. 1. Rough spectrum obtained using FFT of early time information.

TABLE 1 RESONANT FREQUENCIES (MHZ) OF THREE STRUCTURS PREDICTED USING DIFFERERNT METHODS

Model / Method	Empty cavity	RC with a vertical stirrer	Lossy RC with a vertical stirrer
Total least squares MPM	Fail*	Fail	Fail
Proposed method	137.64	137.56	134.39
	217.11	217.24	213.78
Analytical prediction	138.65	-	-
	216.26	-	-

* Fail indicates the results are not reasonable