

Problem 7
Asymmetrical Conductor with a Hole

1. General Description

The model is shown in Fig.1. A thick aluminum plate with a hole, which is placed eccentrically, is set unsymmetrically in a non-uniform magnetic field. The field is produced by the exciting current which varies sinusoidally with time.

The problem is to calculate magnetic fields and eddy currents at various positions.

2. Mesh Description

As the model has no symmetry, the whole region should be discretized. The mesh shown in Fig.2 is recommended. If the discretization of the exciting coil is needed, the mesh shown in Fig.2 should be modified adequately. Though any kind of element is available, an attempt to keep the positions of the nodes and the elements shown in Fig.2 is encouraged.

3. Boundary Conditions

If the boundary conditions are needed, the outermost boundary which is away from the conductor is to be set on the positions shown in Fig.2. On the outermost boundary, the normal component of the flux density is to be equal to zero in order that the flux is parallel to the boundary. Please denote the imposed boundary conditions in Fig.3. In particular, the boundary conditions of corners and edges of the conductor are to be denoted clearly.

4. Input Parameters

Input parameters are denoted in Table 1.

5. Quantities and Distributions to be Presented

To compare results, please complete Tables 2 and 3. B_z at $f=0(\text{Hz})$ in Table 2 means the field produced only by the exciting current. At $\omega t=0(\text{deg})$, the exciting current becomes the maximum.

The following distributions are to be presented at $\omega t=0$ and $90(\text{deg})$.

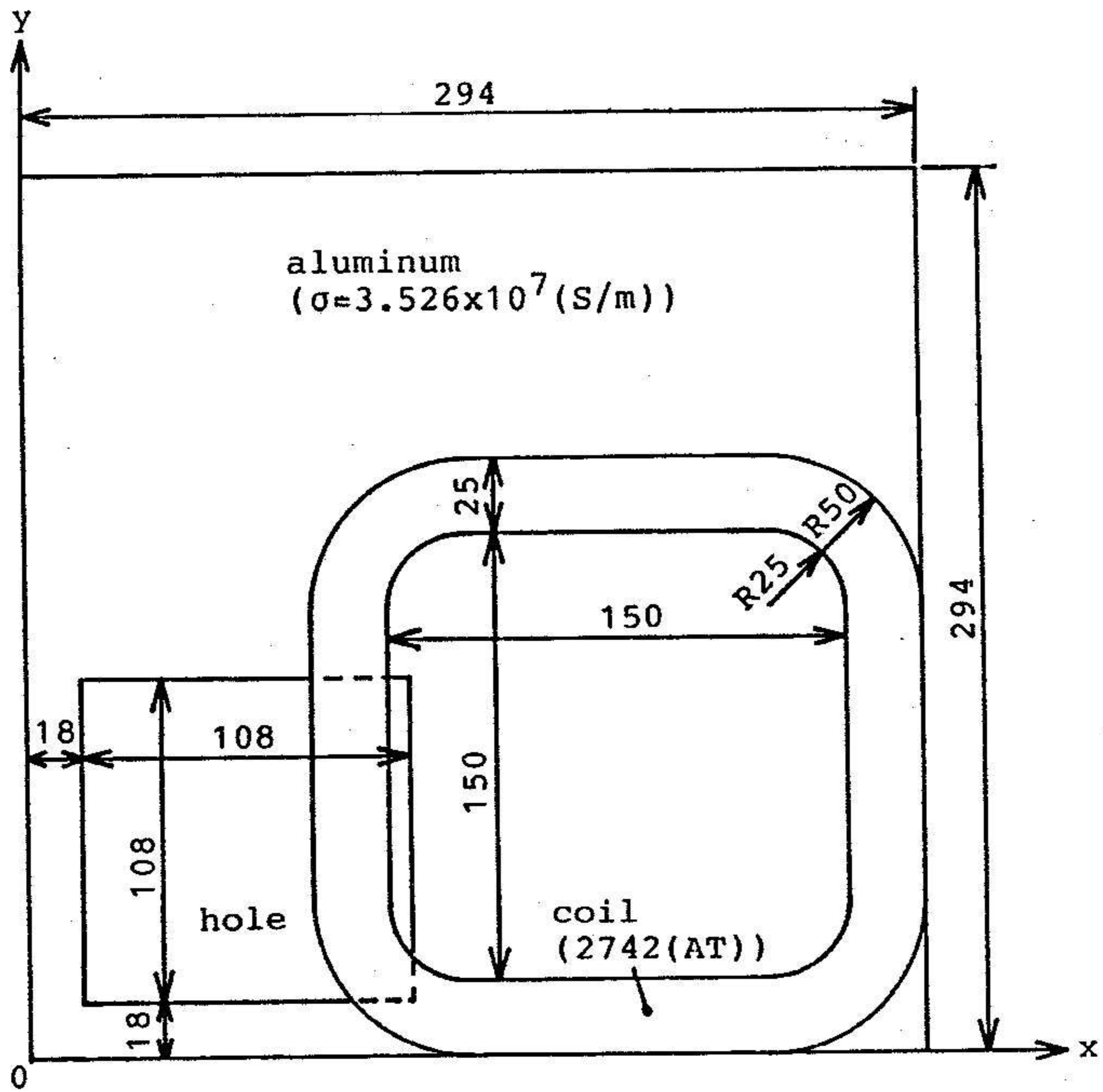
- (1) Distributions of flux density vectors on the $x-z$ plane at $y=72(\text{mm})$.
- (2) Distributions of eddy current vectors on the surfaces of the conductor at $z=19$ and $0(\text{mm})$.
- (3) Distributions of eddy current vectors in the $x-z$ plane between $131 < y < 136(\text{mm})$.

6. Description of Computer Program

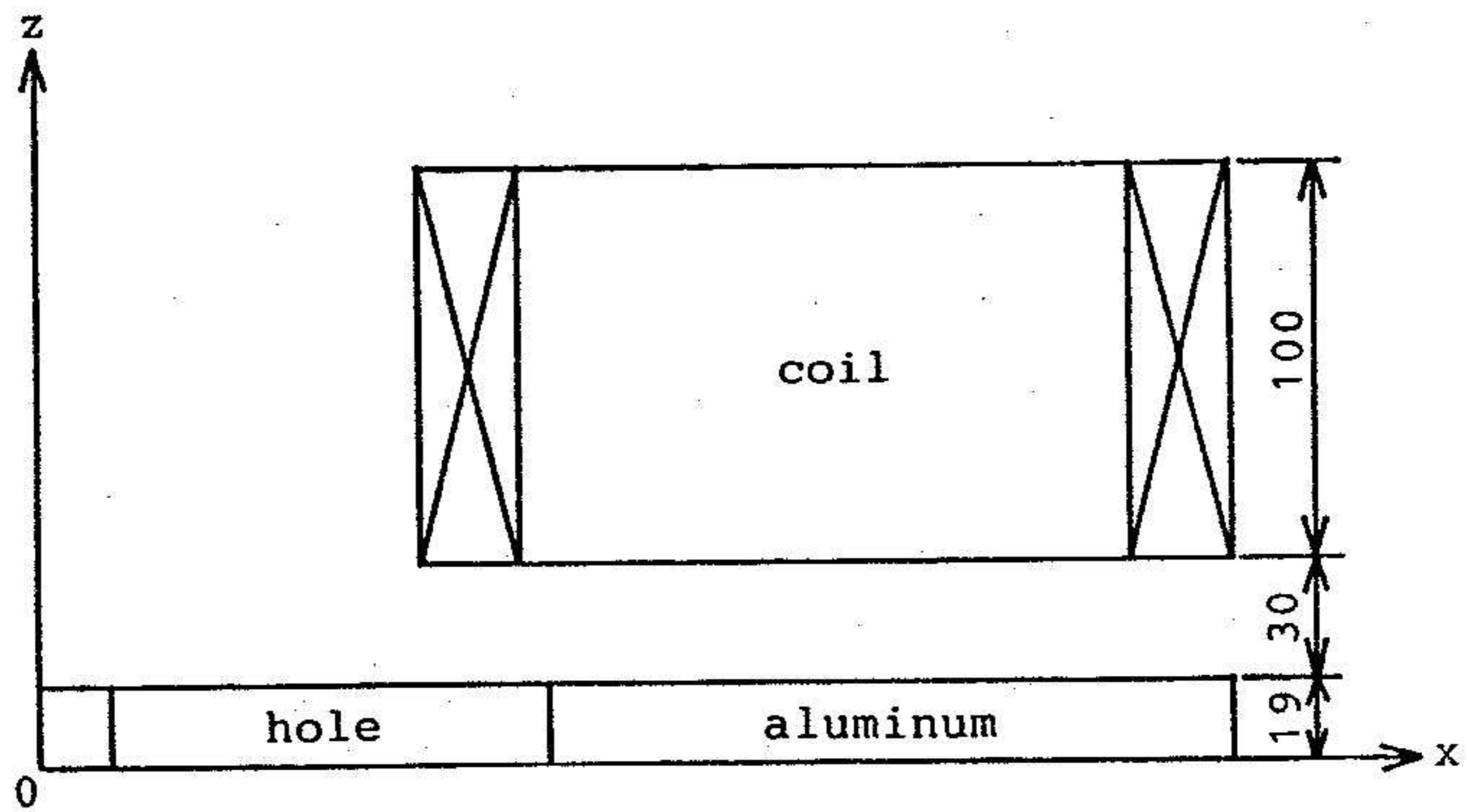
To compare formulations, variables, etc., please complete Table 4. The used memory in the item No.15 is defined as the sum of dimensions declared in the program.

7. References

- [1] D.Rodger & J.F.Eastham : "Multiply Connected Regions in the $A-\psi$ Three-Dimensional Eddy-Current Formulation", IEE Proc. 134, Pt.A, 1, 58 (1987)
- [2] T.Nakata, N.Takahashi, K.Fujiwara & Y.Okada : "Improvements of $T-\Omega$ Method for 3-D Eddy Current Analysis", IEEE Trans. Magnetics, MAG-24, 1 (1988)
- [3] C.R.I.Emson : "Methods for the Solution of Open-Boundary Electromagnetic-Field Problems", IEE Proc. 135, Pt.A, 3, 151 (1988)
- [4] P.Tong & J.N.Rossetos : "Finite-Element Method (Basic Technique and Implementation)", MIT Press (1977)

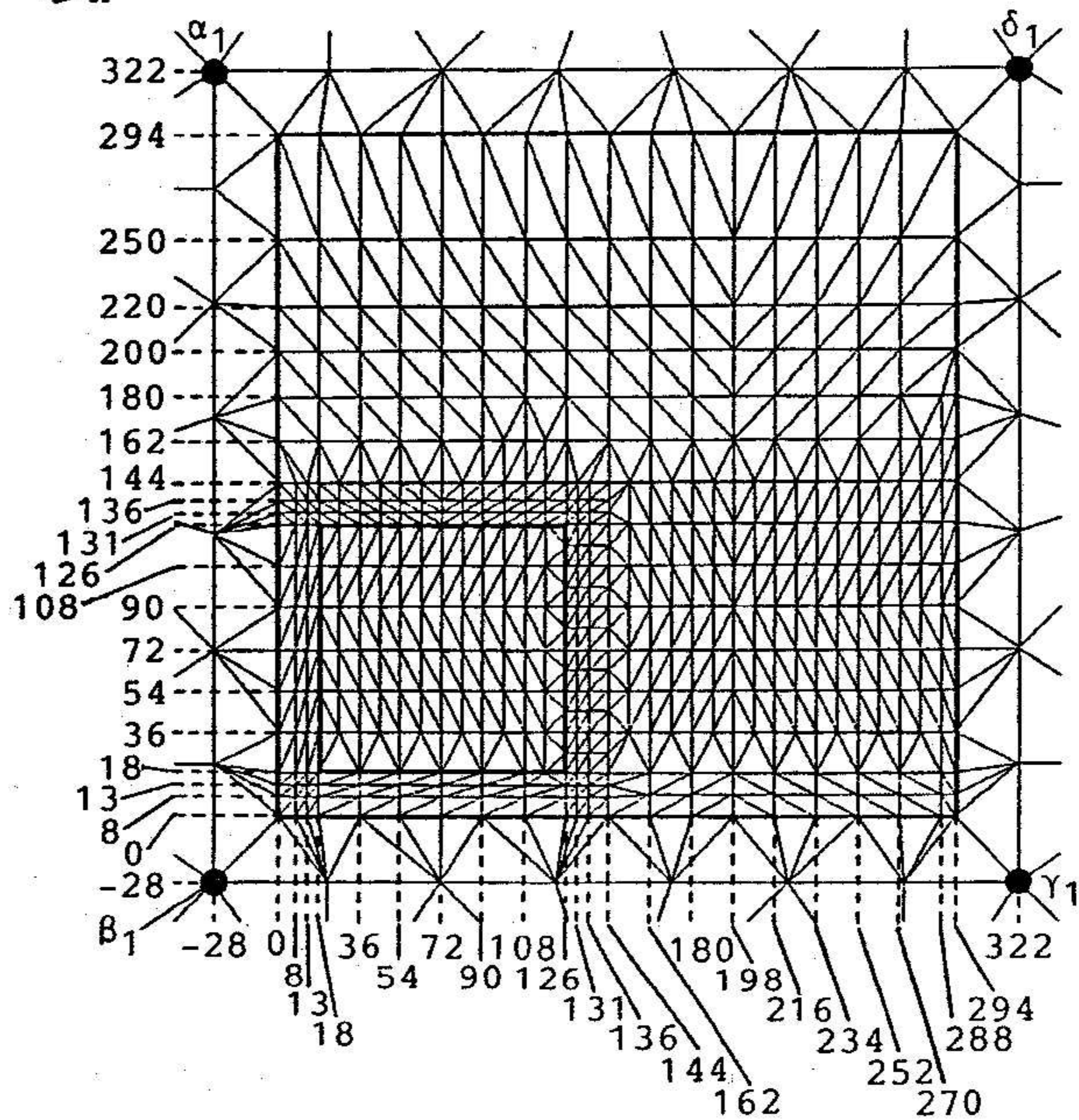
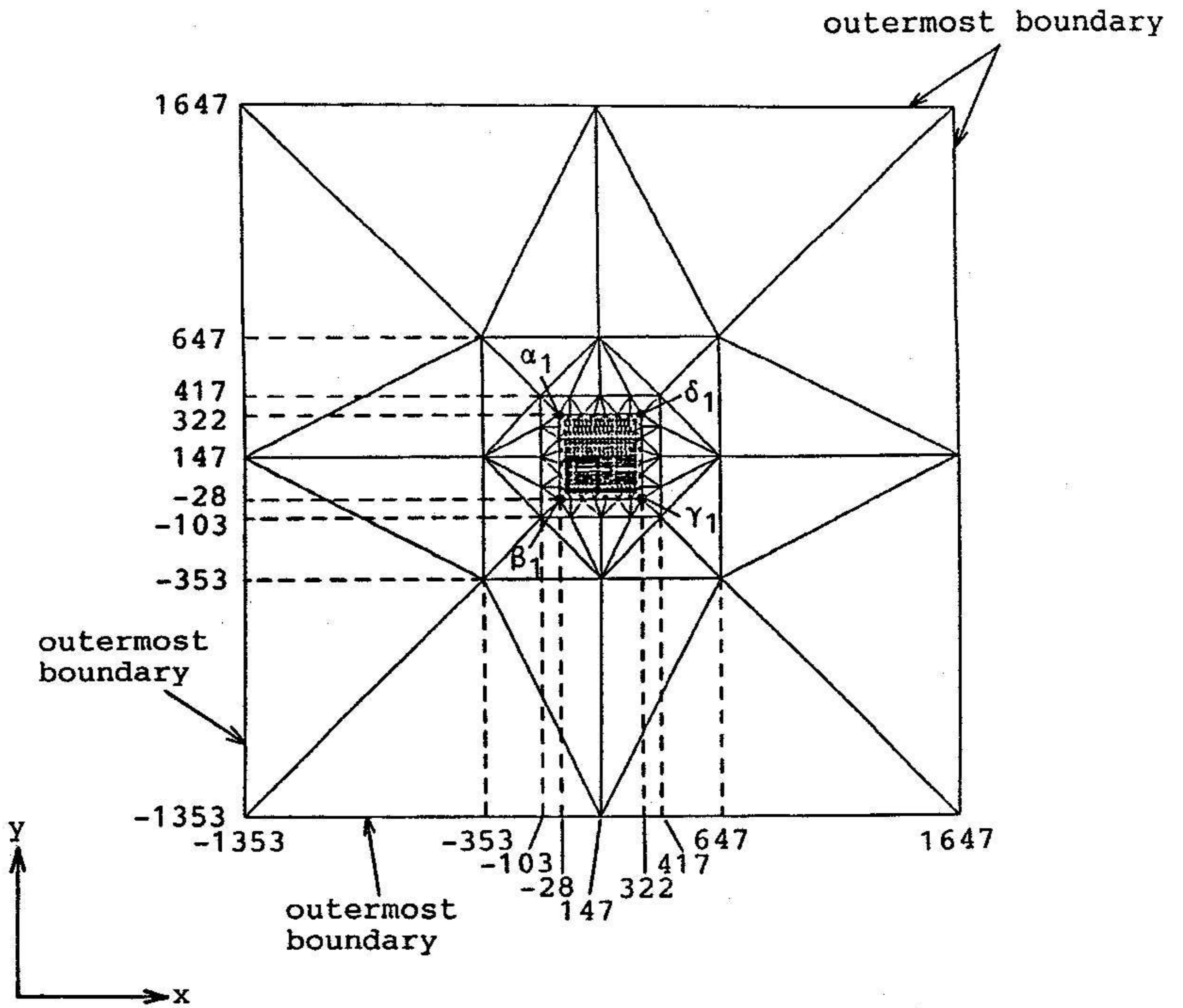


(a) plan



(b) cross-section

Fig.1 Asymmetrical conductor with a hole.



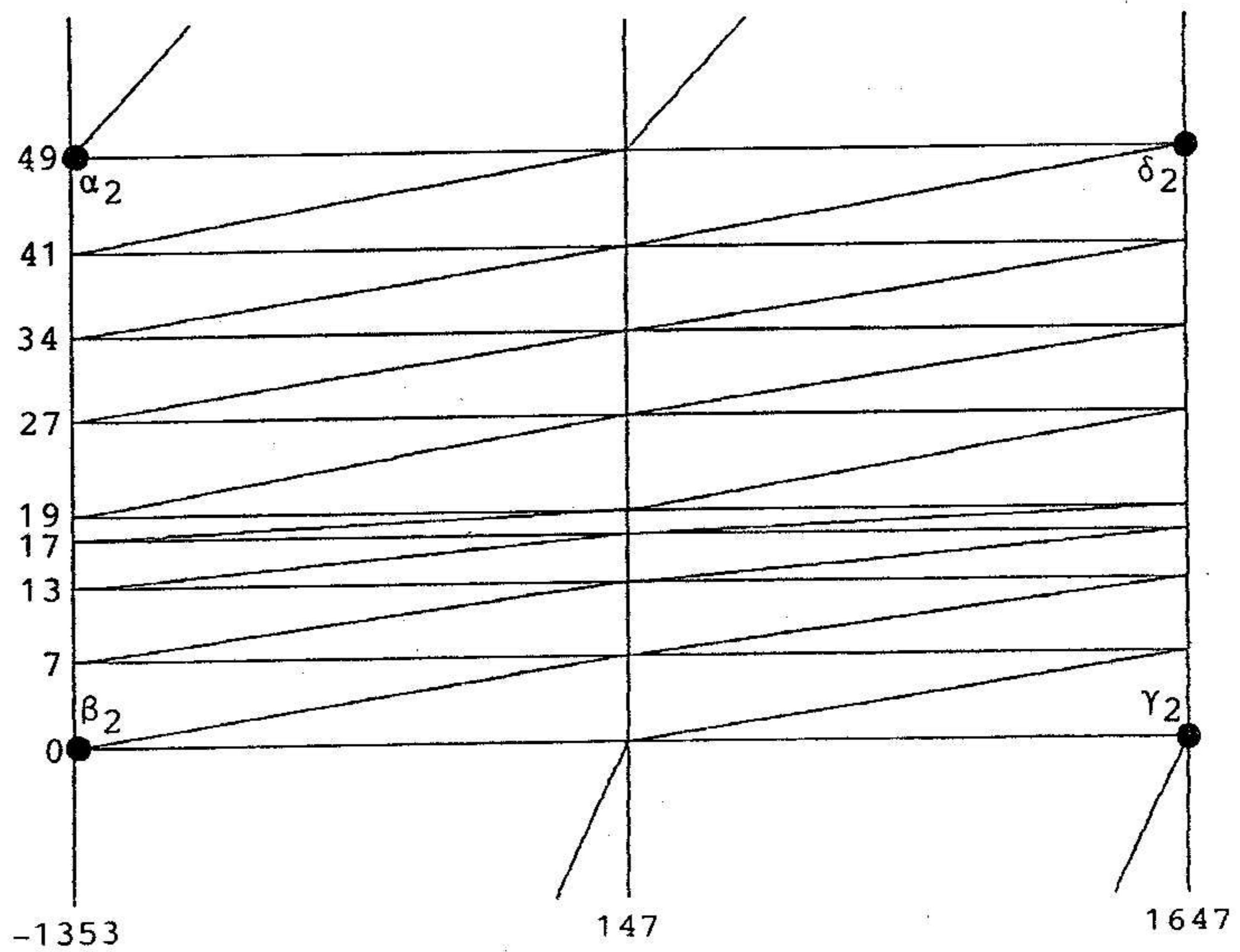
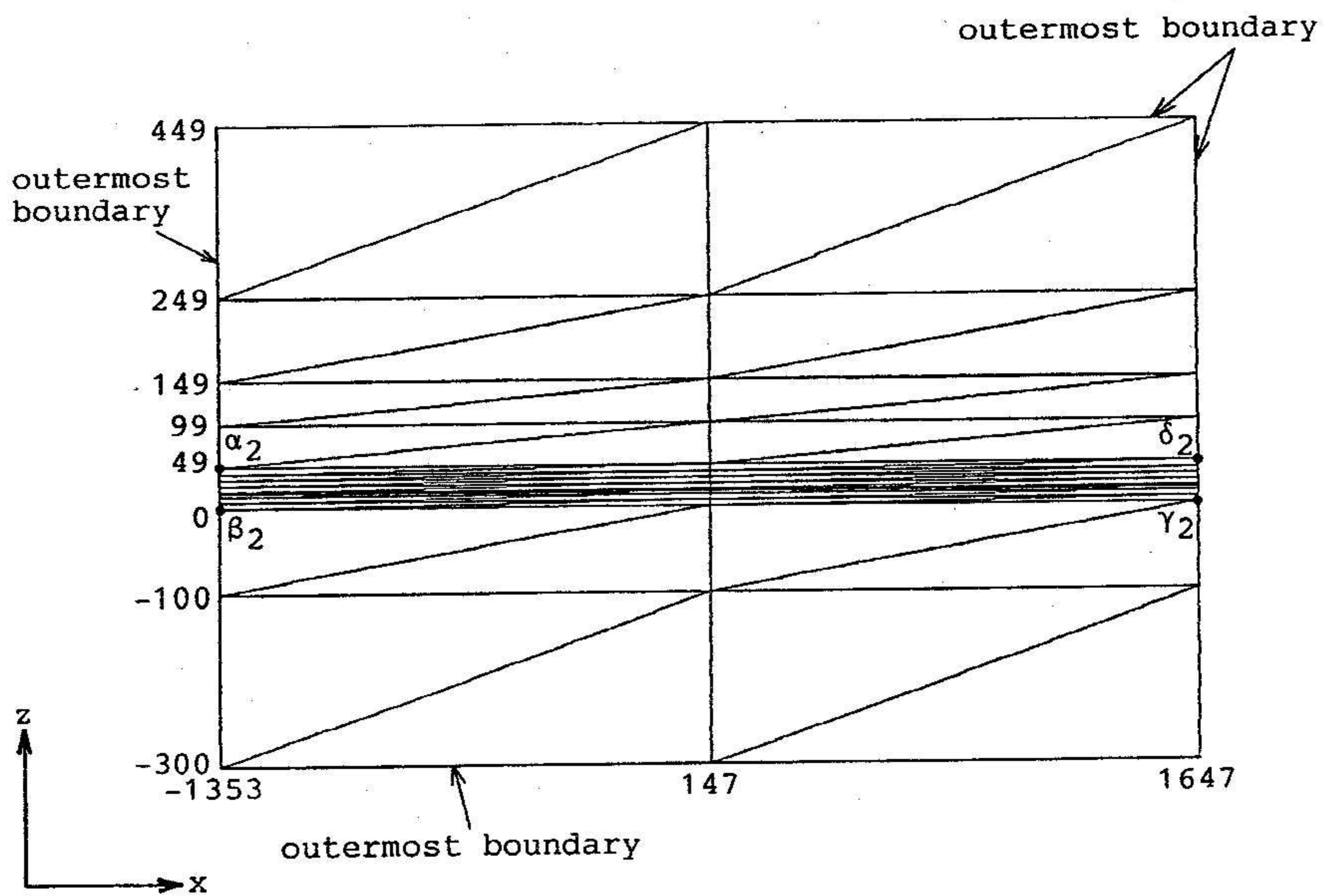
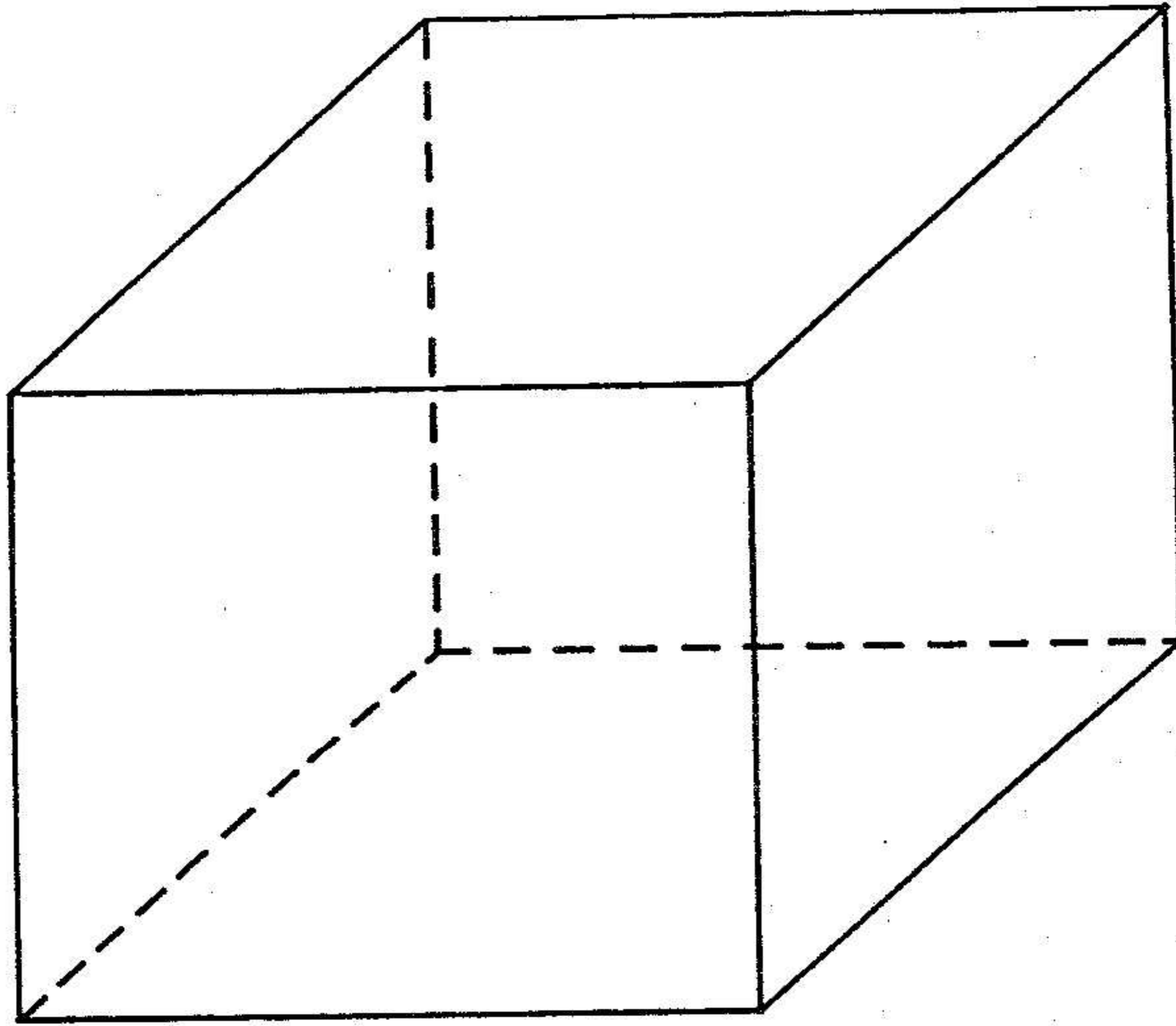
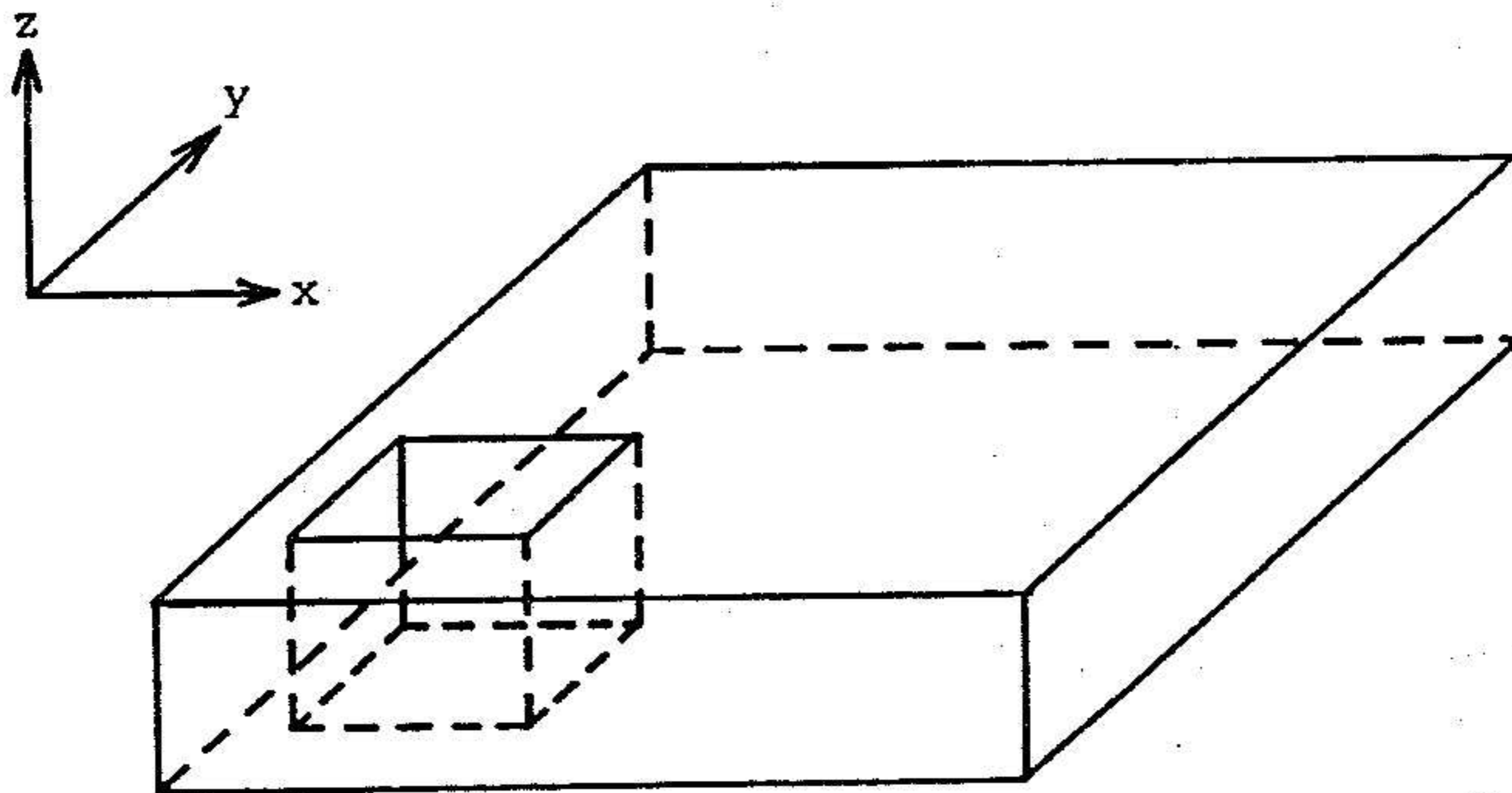


Fig.2(b) Recommended mesh on the x-z plane.

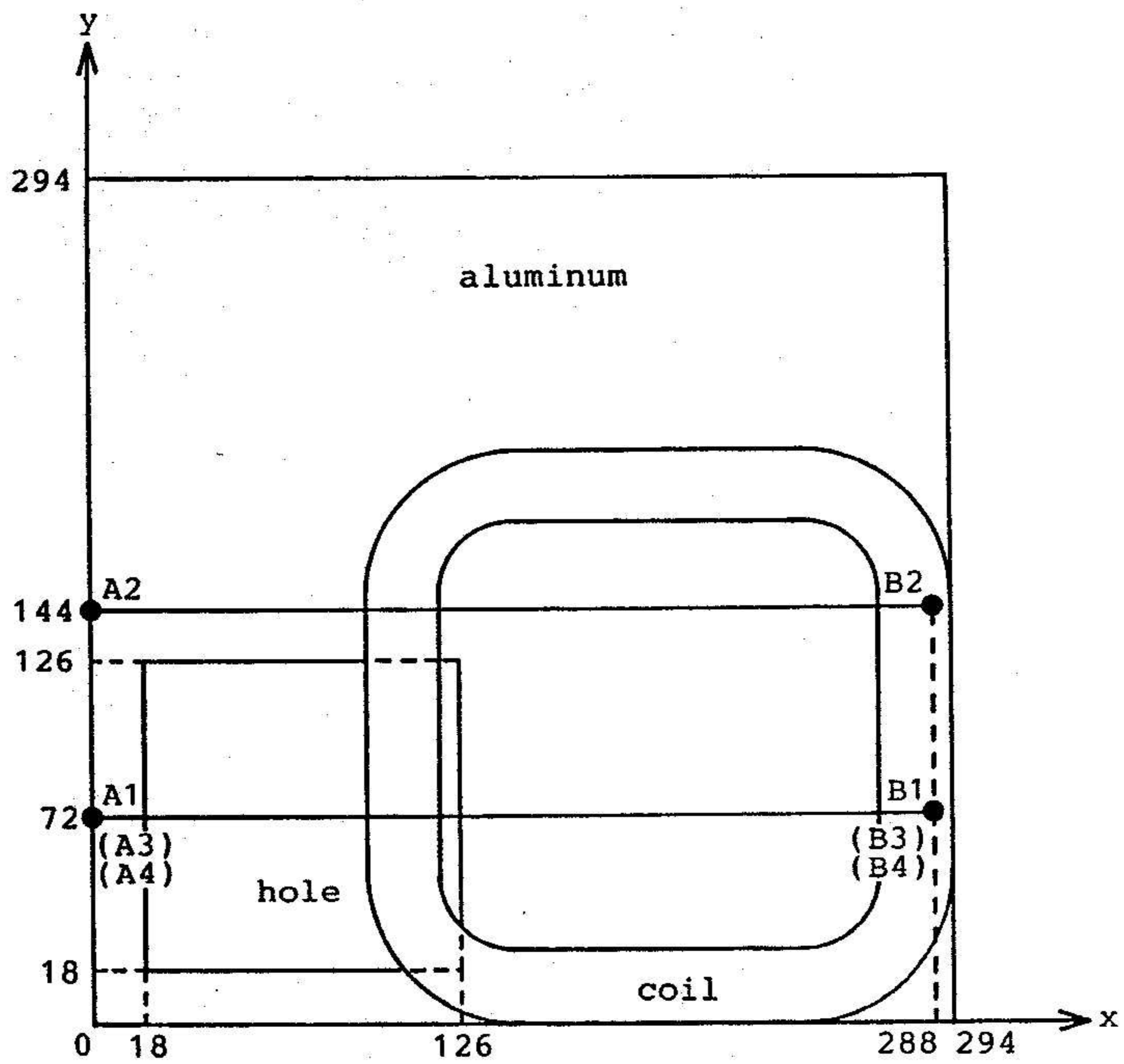


(a) outermost boundary

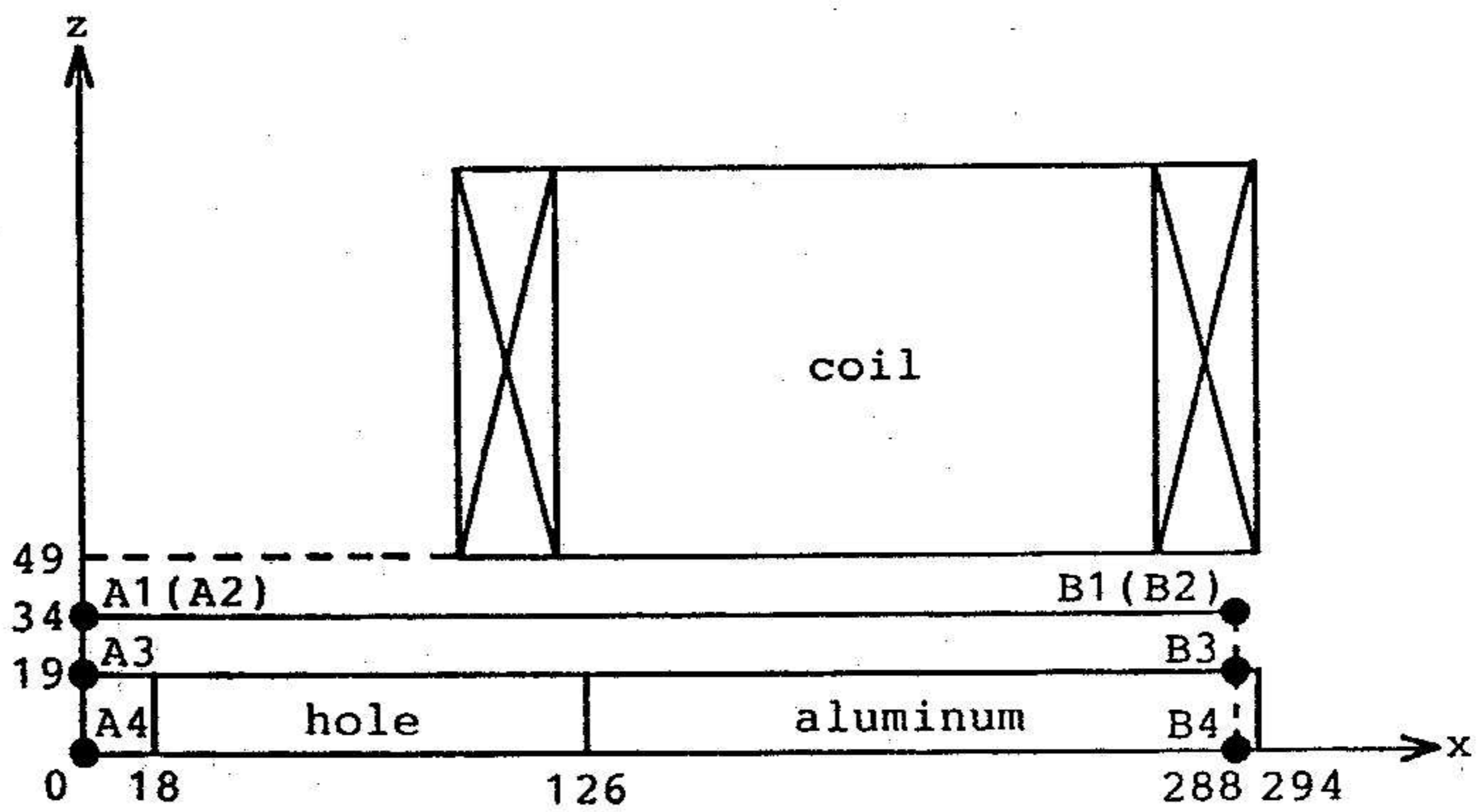


(b) surface of conductor

Fig.3 Boundary conditions.



(a) plan



(b) cross-section

Fig.4 Measured points of the flux density and the eddy current density.

Table 1 Input parameters.

Parameter	Value
Conductivity σ (S/m)	3.526×10^7
Exciting current I_0 (AT)	2742
Frequency f (Hz)	50, 200

Table 2 Z-component B_z (Gauss) of the flux density along the lines A1-B1($y=72, z=34$ (mm)) and A2-B2($y=144, z=34$ (mm)) at the instances of $\omega t=0$ and 90 (deg).

(a) B_z along the line A1-B1($y=72, z=34$ (mm))

No.	x (mm)	f (Hz)				
		0	50		200	
			$\omega t=0$	$\omega t=90$	$\omega t=0$	$\omega t=90$
1	0.0					
2	18.0					
3	36.0					
4	54.0					
5	72.0					
6	90.0					
7	108.0					
8	126.0					
9	144.0					
10	162.0					
11	180.0					
12	198.0					
13	216.0					
14	234.0					
15	252.0					
16	270.0					
17	288.0					

(b) B_z along the line A2-B2($y=144, z=34$ (mm))

No.	x (mm)	f (Hz)			
		50	200		
				$\omega t=0$	$\omega t=90$
1	0.0				
2	18.0				
3	36.0				
4	54.0				
5	72.0				
6	90.0				
7	108.0				
8	126.0				
9	144.0				
10	162.0				
11	180.0				
12	198.0				
13	216.0				
14	234.0				
15	252.0				
16	270.0				
17	288.0				

Table 3 Y-component $J_{ey}(x10^6(A/m^2))$ of the eddy current density along the lines A3-B3($y=72, z=19(mm)$) and A4-B4($y=72, z=0(mm)$) on the surfaces of the conductor at the instances of $\omega t=0$ and $90(deg)$.

(a) J_{ey} along the line A3-B3($y=72, z=19(mm)$)

No.	x(mm)	f (Hz)			
		50		200	
		$\omega t=0$	$\omega t=90$	$\omega t=0$	$\omega t=90$
1	0.0				
2	18.0				
3	36.0				
4	54.0				
5	72.0				
6	90.0				
7	108.0				
8	126.0				
9	144.0				
10	162.0				
11	180.0				
12	198.0				
13	216.0				
14	234.0				
15	252.0				
16	270.0				
17	288.0				

(b) J_{ey} along the line A4-B4($y=72, z=0(mm)$)

No.	x(mm)	f (Hz)			
		50		200	
		$\omega t=0$	$\omega t=90$	$\omega t=0$	$\omega t=90$
1	0.0				
2	18.0				
3	36.0				
4	54.0				
5	72.0				
6	90.0				
7	108.0				
8	126.0				
9	144.0				
10	162.0				
11	180.0				
12	198.0				
13	216.0				
14	234.0				
15	252.0				
16	270.0				
17	288.0				

Table 4 Description of computer program.

No.	Item	Specification
1	Code name	
2	Formulation	<input type="checkbox"/> 1. FEM (Finite Element Method) <input type="checkbox"/> 2. BEM (Boundary Element Method) <input type="checkbox"/> 3. FECM (Finite Element Circuit Method) <input type="checkbox"/> 4. NMM (Network Mesh Method) <input type="checkbox"/> 5. IEM (Integral Equation Method) <input type="checkbox"/> 6. FDM (Finite Difference Method) <input type="checkbox"/> 7. combination (+) <input type="checkbox"/> 8. the others () (Please write references in the item No.16)
3	Governing equations	in conductor
		in vacuum
4	Solution variables	in conductor
		in vacuum
5	Gauge condition	<input type="checkbox"/> 1. not imposed <input type="checkbox"/> 2. imposed <input type="checkbox"/> (a) impose the condition on governing equations directly <input type="checkbox"/> (b) penalty function method <input type="checkbox"/> (c) Lagrange multiplier method <input type="checkbox"/> (d) the others () (Please write references in the item No.16)
6	Technique for treating hole in conductor[1]	<input type="checkbox"/> 1. no special technique <input type="checkbox"/> 2. very low conductivity[2] ($\sigma =$ (S/m)) <input type="checkbox"/> 3. thin artificial conducting sheet (thickness= (mm)) <input type="checkbox"/> 4. cut and potential jump <input type="checkbox"/> 5. the others () (Please write references in the item No.16)
7	Technique for open boundary problem[3]	<input type="checkbox"/> 1. truncation <input type="checkbox"/> 2. mapping <input type="checkbox"/> 3. ballooning <input type="checkbox"/> 4. Zienkiewicz's infinite element <input type="checkbox"/> 5. Tong's infinite element[4] <input type="checkbox"/> 6. BEM or IEM <input type="checkbox"/> 7. the others () (Please write references in the item No.16)

Table 4 Description of computer program (continued).

No.	Item	Specification
8	Magnetic field produced by exciting current	<input type="checkbox"/> 1. Biot-Savart law (analytical) <input type="checkbox"/> 2. Biot-Savart law (numerical) <input type="checkbox"/> 3. taking into account exciting current in governing equations directly
9	Property of coefficient matrix of linear equations	1. symmetric <input type="checkbox"/> (1a) sparse <input type="checkbox"/> (1b) full 2. asymmetric <input type="checkbox"/> (2a) sparse <input type="checkbox"/> (2b) full <input type="checkbox"/> 3. combination ()
10	Solution method for linear equations	<input type="checkbox"/> 1. ICCG <input type="checkbox"/> 2. ICBCG <input type="checkbox"/> 3. ILUBCG <input type="checkbox"/> 4. SOR <input type="checkbox"/> 5. LDL ^T <input type="checkbox"/> 6. LU <input type="checkbox"/> 7. Gauss elimination method <input type="checkbox"/> 8. the others () (Please write references in the item No.16)
11	Element type	<input type="checkbox"/> 1. tetrahedron (nodes, edges) <input type="checkbox"/> 2. triangular (nodes, edges) prism <input type="checkbox"/> 3. hexahedron (nodes, edges) <input type="checkbox"/> 4. triangle (nodes, edges) <input type="checkbox"/> 5. rectangle (nodes, edges) <input type="checkbox"/> 6. the others ()
12	Number of elements	
13	Number of nodes	
14	Number of unknowns	

Table 4 Description of computer program (continued).

No.	Item		Specification
15	Computer	name	
		speed	(MIPS), (MFLOPS)
		main memory (MB)	
		used memory (MB)	
		Precision of data (bits)	
		CPU time (sec)	total
solving linear equations			
16	References on Nos.2 to 10, etc.		

Y-components $J_{ey} (\times 10^6 \text{ A/m}^2)$ of eddy current densities on surface of conductor

(a) line A3-B3 ($y=72\text{mm}, z=19\text{mm}$)

No.	x (mm)	f (Hz)			
		50		200	
		$\omega t=0^\circ$	$\omega t=90^\circ$	$\omega t=0^\circ$	$\omega t=90^\circ$
1	0.0	0.249	-0.629	0.427	-0.623
2	18.0	0.685	-0.873	0.794	-0.755
3	126.0	-0.015	-0.593	1.401	-1.304
4	144.0	-0.103	-0.249	-0.035	-0.229
5	162.0	-0.061	-0.101	0.005	-0.041
6	180.0	-0.004	-0.001	-0.011	-0.014
7	198.0	0.051	0.087	0.007	-0.002
8	216.0	0.095	0.182	0.027	-0.000
9	234.0	0.135	0.322	0.042	0.008
10	252.0	0.104	0.555	0.043	0.033
11	270.0	-0.321	0.822	0.050	0.116
12	288.0	-0.687	0.855	-0.321	0.893

(b) line A4-B4 ($y=72\text{mm}, z=0\text{mm}$)

No.	x (mm)	f (Hz)			
		50		200	
		$\omega t=0^\circ$	$\omega t=90^\circ$	$\omega t=0^\circ$	$\omega t=90^\circ$
1	0.0	0.461	-0.662	1.057	-0.915
2	18.0	0.621	-0.644	1.597	-1.036
3	126.0	1.573	-1.027	4.163	-2.328
4	144.0	0.556	-0.757	1.143	-1.193
5	162.0	0.237	-0.364	0.672	-0.613
6	180.0	0.097	-0.149	0.307	-0.259
7	198.0	-0.034	0.015	-0.050	0.061
8	216.0	-0.157	0.154	-0.370	0.334
9	234.0	-0.305	0.311	-0.749	0.674
10	252.0	-0.478	0.508	-1.205	1.064
11	270.0	-0.660	0.747	-1.575	1.404
12	288.0	-1.217	1.034	-2.583	2.331

Z-components Bz(G) of flux densities

(a) line A1-B1 (y=72mm, z=34mm)

No.	x (mm)	f (Hz)			
		50		200	
		$\omega t=0^\circ$	$\omega t=90^\circ$	$\omega t=0^\circ$	$\omega t=90^\circ$
1	0.0	- 4.90	-1.16	- 3.63	-1.38
2	18.0	-17.88	2.84	-18.46	1.20
3	36.0	-22.13	4.15	-23.62	2.15
4	54.0	-20.19	4.00	-21.59	1.63
5	72.0	-15.67	3.07	-16.09	1.10
6	90.0	0.36	2.31	0.23	0.27
7	108.0	43.64	1.89	44.35	-2.28
8	126.0	78.11	4.97	75.53	-1.40
9	144.0	71.55	12.61	63.42	4.17
10	162.0	60.44	14.15	53.20	3.94
11	180.0	53.91	13.04	48.66	4.86
12	198.0	52.62	12.40	47.31	4.09
13	216.0	53.81	12.05	48.31	3.69
14	234.0	56.91	12.27	51.26	4.60
15	252.0	59.24	12.66	53.61	3.48
16	270.0	52.78	9.96	46.11	4.10
17	288.0	27.61	2.36	24.96	0.98

(b) line A2-B2 (y=144mm, z=34mm)

No.	x (mm)	f (Hz)			
		50		200	
		$\omega t=0^\circ$	$\omega t=90^\circ$	$\omega t=0^\circ$	$\omega t=90^\circ$
1	0.0	- 1.83	-1.63	- 0.86	-1.35
2	18.0	- 8.50	-0.60	- 7.00	-0.71
3	36.0	-13.60	-0.43	-11.58	-0.81
4	54.0	-15.21	0.11	-13.36	-0.67
5	72.0	-14.48	1.26	-13.77	0.15
6	90.0	-5.62	3.40	-6.74	1.39
7	108.0	28.77	6.53	24.63	2.67
8	126.0	60.34	10.25	53.19	3.00
9	144.0	61.84	11.83	54.89	4.01
10	162.0	56.64	11.83	50.72	3.80
11	180.0	53.40	11.01	48.03	4.00
12	198.0	52.36	10.58	47.13	3.02
13	216.0	53.93	10.80	48.25	2.20
14	234.0	56.82	10.54	51.35	2.78
15	252.0	59.48	10.62	53.35	1.58
16	270.0	52.08	9.03	45.37	1.37
17	288.0	26.56	1.79	24.01	0.93